Answer & Solutions

Level-I

1. **(b)**

2. **(d)** $\sin 83° = \cos 7°$

$\therefore$ the given expression is $1 - 1 + 1 = 1$

3. **(c)** $\cot^2 75° = \left(2-\sqrt{3}\right)^3 = 7 - 4\sqrt{3}$

4. **(d)** $\tan\theta = \dfrac{a}{b}$

$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a\tan\theta - b}{a\tan\theta + b} = \frac{a^2-b^2}{a^2+b^2}$$

5. **(d)**

6. **(a)** $\cos^2\theta = 1-\sin^2\theta = 1-\left(\dfrac{11}{15}\right)^2 = \dfrac{225-121}{225} = \dfrac{104}{225}$

$$\Rightarrow \cos\theta = \frac{\sqrt{4\times 26}}{15} = \frac{2\sqrt{26}}{15}$$

7. **(b)** $\sec^2\theta = 1 + \tan^2\theta = 1 + 3 = 4 \Rightarrow \sec\theta = 2$

8. **(c)** $(1 + \tan^2\theta)/\cos^2\theta = \sec^2\theta = 1$

9. **(c)** $\dfrac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta} = \dfrac{4\dfrac{\sin\theta}{\cos\theta} - 1}{4\dfrac{\sin\theta}{\cos\theta} + 1} = \dfrac{4\tan\theta - 1}{4\tan\theta + 1}$

$$= \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$$

10. **(a)** $\dfrac{2\tan 30°}{1+\tan^2 30°} = \dfrac{2\left(\dfrac{1}{\sqrt{3}}\right)}{1+\left(\dfrac{1}{\sqrt{3}}\right)^2} = \dfrac{\dfrac{2}{\sqrt{3}}}{1+\dfrac{1}{3}} = \dfrac{\dfrac{2}{\sqrt{3}}}{\dfrac{4}{3}} = \dfrac{(2)(3)}{(4)(\sqrt{3})}$

$$= \frac{\sqrt{3}}{2} = \sin 60°$$

11. **(d)** $\dfrac{1-\tan^2 45}{1+\tan^2 45} = \dfrac{1-1}{1+1} = \dfrac{0}{2} = 0$

12. **(a)** $\sin 2A = 2\sin A$

This equation in satisfied for $A = 0°$ and for no other value of A given in options i.e., 30°, 45°, 60°.

Because for $A = 0°$

$\sin 2A = \sin 2(0°) = \sin 0° = 0$

and $2\sin A = 2\sin(0°) = 2(0) = 0$

$\therefore$ $\sin 2A = 2\sin A$ for the option $A = 0°$

13. **(c)** $\dfrac{2\tan 30°}{1-\tan^2 30°} = \dfrac{2\left(\dfrac{1}{\sqrt{3}}\right)}{1-\left(\dfrac{1}{\sqrt{3}}\right)^2} = \dfrac{\left(\dfrac{2}{\sqrt{3}}\right)}{1-\dfrac{1}{3}} = \dfrac{\dfrac{2}{\sqrt{3}}}{\dfrac{2}{3}}$

$$= \left(\frac{2}{\sqrt{3}}\right)\left(\frac{3}{2}\right) = \sqrt{3} = \tan 60°$$

14. **(c)** $\cos 9\alpha = \sin\alpha = \cos(90° - \alpha) \Rightarrow 9\alpha = 90° - \alpha$

$\Rightarrow 10\alpha = 90°$

$\Rightarrow 5\alpha = 45° \Rightarrow \tan 5\alpha = \tan 45° = 1$

15. **(b)** $9\sec^2 A - 9\tan^2 A = 9(\sec^2 A - \tan^2 A) = 9(1) = 9$

16. **(c)** $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

$$= \left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right) = \left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right) = \left(\frac{\sin\theta+\sin\theta-1}{\sin\theta}\right)$$

[Use $(a + b)(a - b) = a^2 - b^2$]

$$= \frac{(\cos\theta+\sin\theta)^2-(1)^2}{\cos\theta.\sin\theta}$$

$$= \frac{\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$$

$$= \frac{1+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta} \qquad [\therefore \cos^2\theta + \sin^2\theta = 1]$$

$$= \frac{2\cos\theta\sin\theta}{\cos\theta\sin\theta} = 2$$

17. **(d)** $(\sec A + \tan A)(1 - \sin A)$

$$= \left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$$

$$= \frac{(1-\sin A)}{\cos A}(1-\sin A) = \frac{1-\sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}$$

[$\because$ $\cos^2 A + \sin^2 A = 1 \Rightarrow \cos^2 A = 1 - \sin^2 A$]

$= \cos A$

18. **(d)** $\dfrac{1+\tan^2 A}{1+\cot^2 A} = \dfrac{1+\tan^2 A}{1+\dfrac{1}{\tan^2 A}} = \dfrac{1+\tan^2 A}{\left(\dfrac{\tan^2 A+1}{\tan^2 A}\right)} = \tan^2 A$

19. **(b)** $\sin A/\cos A = \tan A$

20. **(d)** $\tan A = \dfrac{\sin A}{\cos A}$ $\quad\therefore \dfrac{\sin A}{\tan A} = \cos A$

$$\therefore \frac{\sin A}{\tan A} + \cos A = \cos A + \cos A = 2\cos A$$

21. (a) $\cos(40° + x) = \sin 30°$
$\Rightarrow \cos(40° + x) = \cos 60°$
$\Rightarrow 40° + x = 60°$
$\Rightarrow x = 20°$

22. (c) $\sin(A - B) = \frac{1}{2}$ and $\cos(A + B) = \frac{1}{2}$
$\Rightarrow \sin(A - B) = \sin 30°$ and $\cos(A + B) = \cos 60°$
$\Rightarrow (A - B) = 30°$ ..... (1)
and $(A + B) = 60°$ ..... (2)
Solving (1) and (2), we get
$a = 45°$ and $B = 15°$

23. (a) $\tan 1° \tan 2° \tan 3°$ ..... $\tan 89°$
$\Rightarrow (\tan 1°.\tan 89°)(\tan 2° \tan 88°)$ .... $(\tan 44° \tan 46°) \tan 45°$
$\Rightarrow (\tan 1°.\cot 1°)(\tan 2° \tan 2°)$ .... $(\tan 44° \cot 44°) \times 1$
$\Rightarrow 1 \times 1 \times 1 \times$ ..... $\times 1 = 1$

24. (b) We have, $\tan\theta = \frac{x\sin\phi}{1 - x\cos\phi}$

$$\Rightarrow \frac{1 - x\cos\phi}{x\sin\phi} = \frac{1}{\tan\theta} \Rightarrow \frac{1}{x\sin\phi} - \cot\phi = \cot\theta$$

$$\Rightarrow \frac{1}{x\sin\phi} = \cot\theta + \cot\phi$$

25. (a) $\tan^2\theta\sin^2\theta \Rightarrow \frac{\sin^2\theta\sin^2\theta}{\cos^2\theta}$

$$\Rightarrow \sin^2\theta\left(\frac{1-\cos^2\theta}{\cos^2\theta}\right) \Rightarrow \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$$

$\Rightarrow \tan^2\theta - \sin^2\theta$

26. (a) $x = r\sin\alpha\cos\beta, y = r\sin\alpha\sin\beta, z = r\cos\alpha$
$\therefore x^2 + y^2 + z^2 = r^2\sin^2\alpha\cos^2\beta + r^2\sin^2\alpha\sin^2\beta + r^2\cos^2\alpha$
$= r^2\sin^2\alpha(\cos^2\beta + \sin^2\beta) + r^2\cos^2\alpha$
$= r^2\sin^2\alpha + r^2\cos^2\alpha = r^2$

27. (b) $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1, \frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$
Squaring and adding, we get

$$\left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right)^2 + \left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right)^2 = 1 + 1$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$$

28. (c) We have, $\frac{\cos 70°}{\sin 20°} + \frac{\cos 59°}{\sin 31°} - 8\sin^2 30°$

$$\Rightarrow \frac{\sin 20°}{\sin 20°} + \frac{\sin 31°}{\sin 31°} - 8\left(\frac{1}{2}\right)^2$$

[$\because \cos\theta = \sin(90° - \theta), \sin 30° = 1/2$]

$$\Rightarrow 1 + 1 - 8 \times \frac{1}{4} \Rightarrow 2 - 2 = 0$$

29. (a) We have, $\sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$

Now, $3\cos\theta - 4\cos^3\theta$
$\Rightarrow 3\cos 30° - 4\cos^3 30°$

$$\Rightarrow 3 \times \frac{\sqrt{3}}{2} - 4 \times \left(\frac{\sqrt{3}}{2}\right)^2 \Rightarrow \frac{3\sqrt{3}}{2} - \frac{4 \times 3\sqrt{3}}{8}$$

$$\Rightarrow \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$$

30. (c) We have, $\tan\theta + \cot\theta = 2$
$\Rightarrow (\tan\theta + \cot\theta)^2 = (2)^2$
$\Rightarrow \tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 4$
$\Rightarrow \tan^2\theta + \cot^2\theta + 2 = 4$
$\Rightarrow \tan^2\theta + \cot^2\theta = 2$

31. (c) We have, $\cos(81° + \theta) = \sin\left(\frac{k}{3} - \theta\right)$

$$\Rightarrow 81° + \theta + \frac{k}{3} - \theta = 90°$$

[$\because \cos\alpha = \sin\beta$, then $\alpha + \beta = 90°$]

$$\Rightarrow \frac{k}{3} = 90° - 81° \Rightarrow k = 27°$$

32. (b) We have, $\tan x = \sin 45° \cos 45° + \sin 30°$

$$\Rightarrow \tan x = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2}$$

$$\Rightarrow \tan x = \frac{1}{2} + \frac{1}{2} = 1 \Rightarrow x = 45°$$

33. (d) We have,
$\cos^2 5° + \cos^2 10° +$ ..... $+ \cos^2 85° + \cos^2 90°$
$= (\cos^2 5° + \cos^2 85°) + (\cos^2 10° + \cos^2 80°)$ ..... $\cos^2 45° + \cos^2 90°$
$= (\cos^2 5° + \cos^2 5°) + (\cos^2 10° + \sin^2 10°)$ ..... $\left(\frac{1}{\sqrt{2}}\right)^2 + 0$

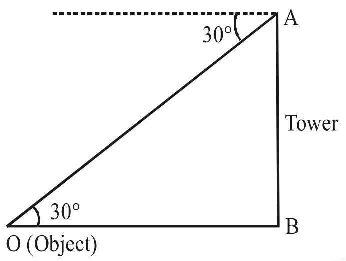
$$= (1 + 1 + \text{..... 8 times}) + \frac{1}{2} = 8 + \frac{1}{2} = \frac{17}{2}$$

34. (b) We have, $\frac{\sin(90° - \theta)\sin\theta}{\tan\theta} + \sin^2\theta$

$$= \frac{\cos\theta\sin\theta}{\frac{\sin\theta}{\cos\theta}} + \sin^2\theta = \cos^2\theta + \sin^2\theta = 1$$

35. (d) **Hint:** $\sin 30° = \frac{BC}{AC} \Rightarrow \frac{1}{2} = \frac{BC}{6\text{ cm}} \Rightarrow BC = 3$ cm.

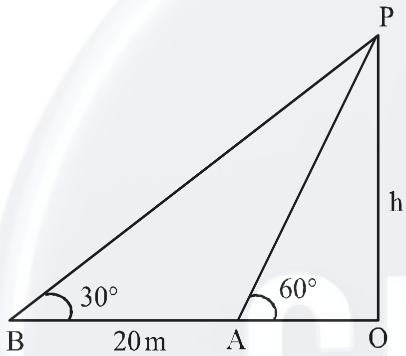
36. **(c)** **Hint:** $\tan 30° = \dfrac{AB}{OB}$



$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{75\text{ m}}{OB}$

$\Rightarrow OB = 75\sqrt{3}$ m

37. **(c)**



OA = $h \cot 60°$, OB = $h \cot 30°$

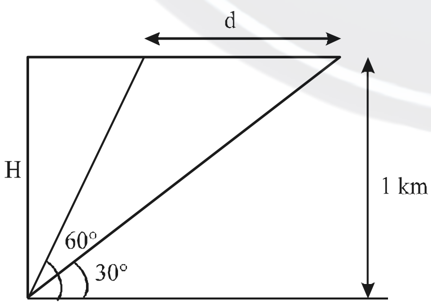
OB – OA = 20 = h (cot 30° – cot 60°)

$$\Rightarrow h = \frac{20}{\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)} = \frac{20\sqrt{3}}{2} = 10\sqrt{3}$$

38. **(b)** d = H cot 30° – H cot 60°

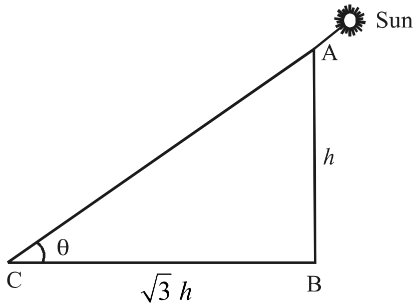
Time taken = 10 second

$$\text{speed} = \frac{\cot 30° - \cot 60°}{10} \times 60 \times 60 = 240\sqrt{3}$$



39. **(b)**

40. **(b)** **Hint :** Let height of tower (AB) be h metres, then length of its shadow $(BC) = \sqrt{3}\,h$ metres.



Let angle of elevation be θ,

then $\tan\theta = \dfrac{h}{\sqrt{3}h} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30°$

41. **(a)**

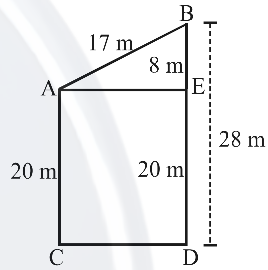
42. **(c)** $AE^2 = AB^2 - BE^2$

$= 17^2 - 8^2$

$= 289 - 64 = 225$

$\Rightarrow$ AE = 15 m

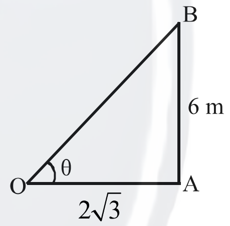
$\Rightarrow$ CD = 15 m



43. **(a)** $\dfrac{6}{2\sqrt{3}} = \tan\theta$

$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \theta = 60°$



44. **(a)** Length of the string of the kite AB = 85 m

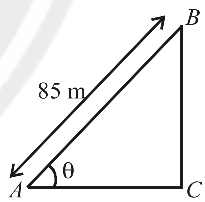
$\tan\theta = \dfrac{15}{8}$

$\Rightarrow \cot\theta = \dfrac{8}{15}$

$\Rightarrow \operatorname{cosec}^2\theta - 1 = \dfrac{64}{225}$

$\Rightarrow \operatorname{cosec}^2\theta = 1 + \dfrac{64}{225} = \dfrac{289}{15}$

$\Rightarrow \operatorname{cosec}\theta = \sqrt{\dfrac{289}{15}} = \dfrac{17}{15} \Rightarrow \sin\theta = \dfrac{15}{17}$



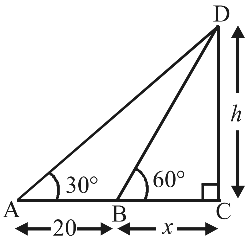
In ΔABC, $\sin\theta = \dfrac{BC}{AB}$

$\Rightarrow \dfrac{15}{17} = \dfrac{BC}{85} \Rightarrow BC = 75\text{cm}$

$\therefore$ Height of kite = 75 cm

45. **(b)** Suppose height of the chimney is h metre. Let A and B be the point of observation and distance be = x m.

In ΔACD,



$$\tan 30° = \frac{CD}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20+x}$$

$$\Rightarrow 20 + x = h\sqrt{3}$$

$$\Rightarrow x = h\sqrt{3} - 20 \quad \text{..... (1)}$$

Now, in ΔABC, $\tan 45° = \frac{CD}{BC}$

$$\Rightarrow 1 = \frac{h}{x} \Rightarrow x = h \quad \text{..... (2)}$$

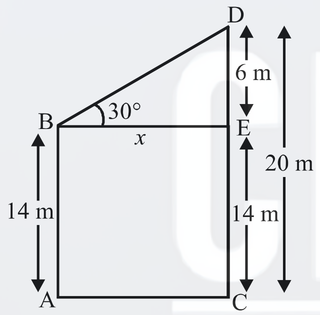
From (1) and (2), we get

$$h = h\sqrt{3} - 20 \Rightarrow h\sqrt{3} - h = 20$$

$$\therefore h = \frac{20}{\sqrt{3}-1}$$

46. **(a)** Here, CD = 20 m [Height of big pole]

AB = 14 m [Height of small pole]



DE = CD – CE ⇒ DE = CD – AB [∵ AB = CE]

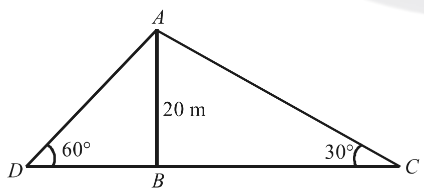
⇒ DE = 20 – 14 = 6 m

In ΔBDE, $\sin 30° = \frac{DE}{BD}$

$$\Rightarrow \frac{1}{2} = \frac{6}{BD} \Rightarrow BD = 12 \text{ m}$$

∴ Length of wire = 12 m

47. **(a)** C and D be the position of the men and AB is the height of flagstaff.



In ΔABC, $\tan 30° = \frac{AB}{BC}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{20}{BC} \Rightarrow BC = 20\sqrt{3}$$

In ΔABD, $\tan 60° = \frac{AB}{BD}$

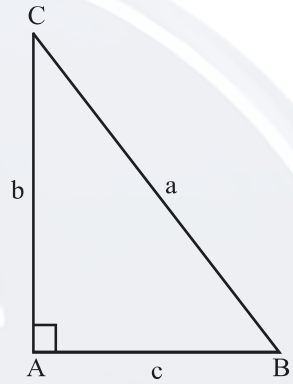
$$\Rightarrow \sqrt{3} = \frac{20}{BD} \Rightarrow BD = \frac{20}{\sqrt{3}}$$

Distance between the men, CD = BC + BD

$$= 20\sqrt{3} + \frac{20}{\sqrt{3}} + \frac{60+20}{\sqrt{3}} = \frac{80}{\sqrt{3}}$$

$$= \frac{80\sqrt{3}}{3} = \frac{80 \times 1.73}{3} = 46.19 \text{ m}$$

48. **(b)**



In right angled ΔABC,

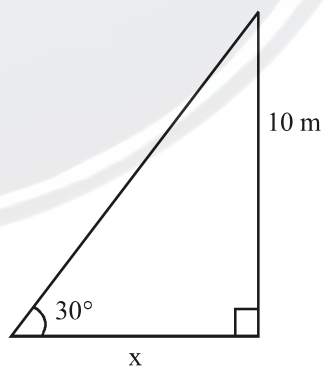
$$\tan \text{B} = \frac{\text{P}}{\text{B}} = \frac{\text{b}}{\text{c}}$$

$$\tan \text{C} = \frac{\text{P}}{\text{B}} = \frac{\text{c}}{\text{b}}$$

$$\tan \text{B} + \tan \text{C} = \frac{\text{b}}{\text{c}} + \frac{\text{c}}{\text{b}}$$

$$= \frac{\text{b}^2 + \text{c}^2}{\text{bc}} = \frac{\text{a}^2}{\text{bc}} \quad [\because \text{a}^2 = \text{b}^2 + \text{c}^2]$$

49. **(a)**



Let ‘x’ be the distance of foot of ladder

$$\tan 30° = \frac{\text{P}}{\text{B}} = \frac{10}{\text{x}}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{\text{x}} \Rightarrow \text{x} = 10\sqrt{3} \text{ m}$$

50. (d) $\tan 7^\circ \tan 23^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ$

$= \tan(90^\circ - 83^\circ)\tan(90^\circ - 67^\circ)\tan 60^\circ \tan 67^\circ \tan 83^\circ$

$= \cot 83^\circ \cot 67^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ$

$[\because \tan(90^\circ - \theta) = \cot\theta]$

$$= \frac{1}{\tan 83^\circ} \times \frac{1}{\tan 67^\circ} \times \tan 60^\circ \times \tan 67^\circ \times \tan 83^\circ$$

$= \tan 60^\circ = \sqrt{3}$

51. (c) $(\sec\theta - \cos\theta)(\csc\theta - \sin\theta)(\tan\theta + \cot\theta)$

$$\Rightarrow \left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$\Rightarrow \frac{1-\cos^2\theta}{\cos\theta} \times \frac{1-\sin^2\theta}{\sin\theta} \times \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$\Rightarrow \frac{\sin^2\theta}{\cos\theta} \times \frac{\cos^2\theta}{\sin\theta} \times \frac{1}{\sin\theta\cos\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$\Rightarrow 1$

52. (b) $\tan(\theta_1 + \theta_2) = \sqrt{3}$

or $\tan(\theta_1 + \theta_2) = \tan 60^\circ$

$\theta_1 + \theta_2 = 60^\circ \quad \ldots (1)$

$$\sec(\theta_1 - \theta_2) = \frac{2}{\sqrt{3}}$$

or $\sec(\theta_1 - \theta_2) = \sec 30^\circ \quad \ldots (2)$

$\theta_1 - \theta_2 = 30^\circ$

Adding equations (1) & (2)

$\theta_1 + \theta_2 + \theta_1 - \theta_2 = 90^\circ$

$\theta_1 = 45^\circ$ & $\theta_2 = 15^\circ$

Now, $\sin 2 \times 45^\circ + \tan 3 \times 15^\circ$

$= \sin 90^\circ + \tan 45^\circ = 1 + 1 = 2$

53. (a) $\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots \tan 89^\circ$

$= \tan 1^\circ \tan 2^\circ \ldots \tan 45^\circ \ldots \tan(90 - 2)\tan(90 - 1)$

$= \tan 1^\circ \tan 2^\circ \ldots 1 \ldots \cot 2^\circ \cot 1^\circ$

$= (\tan 1^\circ \cot 1^\circ)(\tan 2^\circ \cot 2^\circ) \ldots 1$

$= 1$

54. (a) $4\tan^2\theta + 9\cot^2\theta$

$\Rightarrow (2\tan\theta)^2 + (3\cot\theta)^2$

$(2\tan\theta)^2 + (3\cot\theta)^2 - 12 + 12$

$= (2\tan\theta - 3\cot\theta)^2 + 12$

$\therefore$ Minimum value $= 12$ because $(2\tan\theta - 3\cot\theta)^2 \geq 0$

55. (c) $\sin\theta - \cos\theta = \dfrac{1}{2}$

$\sin\theta + \cos\theta = x.$

On squaring and adding.

$$2(\sin^2\theta + \cos^2\theta) = \frac{1}{4} + x^2$$

$$\Rightarrow x^2 = 2 - \frac{1}{4} = \frac{7}{4}$$

$$\Rightarrow x = \frac{\sqrt{7}}{2}$$

56. (c) $\csc\theta - \cot\theta = \dfrac{7}{2} \quad \ldots (1)$

$\csc^2\theta - \cot^2\theta = 1$

$\Rightarrow (\csc\theta + \cot\theta)(\csc\theta - \cot\theta) = 1$

$$\Rightarrow \csc\theta + \cot\theta = \frac{1}{\text{cosec} - \cot\theta} = \frac{2}{7} \quad \ldots (2)$$

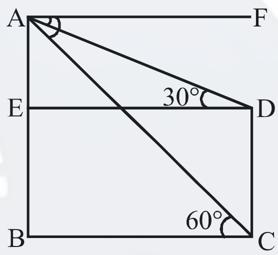
On adding both equations.

$$2\csc\theta = \frac{7}{2} + \frac{2}{7}$$

$$= \frac{49 + 4}{14} = \frac{53}{14}$$

$$\Rightarrow \csc\theta = \frac{53}{28}$$

57. (c)



AB = Height of Hill = 200 metre

$\angle ADE = 30^\circ$

$\angle ACB = 60^\circ$

DE = BC = x metre

From ΔABC, $\tan 60^\circ = \dfrac{AB}{BC}$

$$\Rightarrow \sqrt{3} = \frac{200}{x} \Rightarrow x = \frac{200}{\sqrt{3}} \text{ metre}$$

From ΔAED,

$$\tan 30^\circ = \frac{AE}{DE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AE}{\frac{200}{\sqrt{3}}} \Rightarrow AE = \frac{200}{3} \text{ meter}$$

$$\therefore CD = 200 - \frac{200}{3} = \frac{400}{3} = 133\frac{1}{3} \text{ metre}$$

58. (b) $\sec^2\theta + \tan^2\theta = 7$

$1 + \tan^2\theta + \tan^2\theta = 7$

$$\tan^2\theta = \frac{6}{2} = 3$$

$$\text{for } 0 \leq \theta \leq \frac{\pi}{2} \quad \tan\theta = \sqrt{3}$$

$$\theta = 60^\circ = \frac{\pi}{3}$$

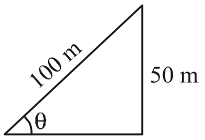
59. **(c)** $\sin^2x + \cos^2x - 2(\sec^2x - \tan^2x)$

$1 - 2(1) = -1$

60. **(a)** $\dfrac{a^2}{a^2\sin^2\theta} - \dfrac{b2}{b^2\tan^2\theta}$

$\text{cosec}^2\theta - \cot^2\theta = 1$

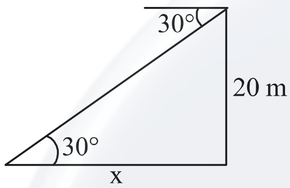
61. **(d)**



$\sin\theta = \dfrac{50\,\text{m}}{100\,\text{m}} = \dfrac{1}{2}$

$\theta = 30°$

62. **(b)**



$\tan 30° = \dfrac{20}{x}$

$\dfrac{1}{\sqrt{3}} = \dfrac{20}{x}$

$x = 20\sqrt{3}$

63. **(a)** $\dfrac{2\sin\theta - \cos\theta}{\cos\theta + \sin\theta} = 1$

Dividing numerator and denominator by $\cos\theta$.

$$\frac{\dfrac{2\sin\theta}{\cos\theta} - \dfrac{\cos\theta}{\cos\theta}}{\dfrac{\cos\theta}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}} = 1$$

$\Rightarrow \dfrac{2\tan\theta - 1}{1 + \tan\theta} = 1$

$\Rightarrow 2\tan\theta - 1 = 1 + \tan\theta \Rightarrow \tan\theta = 2$

Hence, $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{2}$

64. **(c)** $\tan\left(\dfrac{\pi}{2} - \dfrac{\theta}{2}\right) = \sqrt{3}$

$\tan\left(\dfrac{\pi}{2} - \dfrac{\theta}{2}\right) = \tan\dfrac{\pi}{3}$

$\dfrac{\pi}{2} - \dfrac{\theta}{2} = \dfrac{\pi}{3} \Rightarrow \dfrac{\theta}{2} = \dfrac{\pi}{2} - \dfrac{\pi}{3} \Rightarrow \theta = \dfrac{\pi}{3}$

$\therefore \cos\theta = \cos\dfrac{\pi}{3} = \dfrac{1}{2}$

65. **(c)** $\dfrac{2}{\sin^2 23°}.\dfrac{\sin^2 23°}{\cos^2 23°} - \left(\sin^2 23 + \cos^2 23°\right) - \tan^2 23°$

$= 2\sec^2 23° - 1 - \tan^2 23°$

$= (\sec^2 23° - 1) + (\sec^2 23° - \tan^2 23°)$

$= \tan^2 23° + 1 = \sec^2 23°$

66. **(d)** $\dfrac{\dfrac{5\sin\theta - 3\cos\theta}{\cos\theta}}{\dfrac{5\sin\theta + 2\cos\theta}{\cos\theta}} = \dfrac{5\tan\theta - 3}{5\tan\theta + 2} = \dfrac{5\times\dfrac{4}{5} - 3}{5\times\dfrac{4}{5} + 2}$

$= \dfrac{1}{6}$

67. **(d)** $x\sin\theta + y\cos\theta = \sqrt{x^2 + y^2}$

Put $x = \sin\theta$

$y = \cos\theta$ in the above equation, we have

$\sin^2\theta + \cos^2\theta = \sqrt{\sin^2\theta + \cos^2\theta}$

$\Rightarrow 1 = 1$

$\Rightarrow x = \sin\theta$ & $y = \cos\theta$ is the solution of above equation. Now, on using $x = \sin\theta$ & $y = \cos\theta$ in

$\dfrac{\cos^2\theta}{a^2} + \dfrac{\sin^2\theta}{b^2} = \dfrac{1}{x^2 + y^2}$

$\Rightarrow \dfrac{y^2}{a^2} + \dfrac{x^2}{b^2} = \dfrac{1}{\sin^2\theta + \cos^2\theta}$

$\Rightarrow \dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$

68. **(a)** $2\cos 2\theta = 1$

$\cos 2\theta = \dfrac{1}{2}$

$\theta = 30°$

$\cot 30° = \sqrt{3}$

69. **(d)** $\cos^2\theta = \dfrac{(x+y)^2}{4xy}$

$1 - \cos^2\theta = 1 - \dfrac{(x+y)^2}{4xy} = -\dfrac{(x-y)^2}{4xy}$

$\sin^2\theta$ cannot be – ve

Both $\sin^2\theta$ and $\cos^2\theta$ will be +ve when $x = y$

70. **(a)** Given $\tan^2\theta - \sin^2\theta = x$

$\dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta = x \Rightarrow \sin^2\theta\left(\dfrac{1}{\cos^2\theta} - 1\right) = x$

$\dfrac{\sin^2\theta(1 - \cos^2\theta)}{\cos^2\theta} \Rightarrow \dfrac{\sin^2\theta}{\cos^2\theta}\left(1 - \cos^2\theta\right) = x$

or $\boxed{\tan^2\theta\sin^2\theta = x}$

71. (a) $\dfrac{\sin 25° \cos 65° + \cos 25° \sin 65°}{\tan^2 70° - \sec^2 70°}$

$= \dfrac{\sin(25° + 65°)}{-1} = \dfrac{\sin 90°}{(-1)} = -1$

72. (b) $4\cos^2\theta - 4\cos\theta + 1 = 0$

$(2\cos\theta - 1)^2 = 0$

or, $2\cos\theta = 1$

$\Rightarrow \cos\theta = \dfrac{1}{2}$

$\theta = 60°$

Hence, the value of $\tan(\theta - 15°) = \tan(60° - 15°)$

$= \tan 45° = 1$

73. (a) $\left(r\cos\theta - \sqrt{3}\right)^2 + (r\sin\theta - 1)^2 = 0$

If and only if, $r\cos\theta - \sqrt{3} = 0$, and $r\sin\theta - 1 = 0$

$r\cos\theta = \sqrt{3}$ ...(1)

$r\sin\theta = 1$ ...(2)

dividing (1) by (2)

$\tan\theta = \dfrac{1}{\sqrt{3}}$

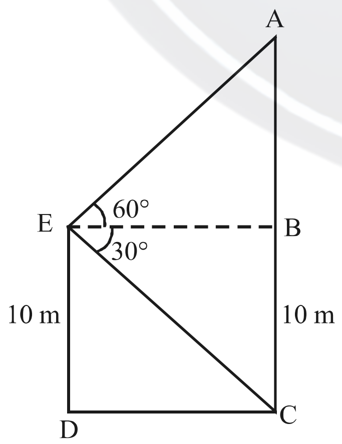
$\sec\theta = \sqrt{1 + \tan^2\theta} = \sqrt{1 + \dfrac{1}{3}} = \dfrac{2}{\sqrt{3}}$

Also, $r^2\cos^2\theta + r^2\sin^2\theta = \left(\sqrt{3}\right)^2 + 1^2$

$r^2 = 4 \Rightarrow r = 2$

$$\frac{r\tan\theta + \sec\theta}{r\sec\theta + \tan\theta} = \frac{2 \times \frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}}{2 \times \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}} = \frac{4/\sqrt{3}}{5/\sqrt{3}} = \frac{4}{5}$$

74. (c)



ED = BC = 10 m

In ΔABE, $\tan 60° = \dfrac{AB}{EB}$

$\sqrt{3} = \dfrac{AB}{EB} \Rightarrow AB = \sqrt{3}EB$ ...(1)

In ΔEBC, $\tan 30° = \dfrac{BC}{EB}$

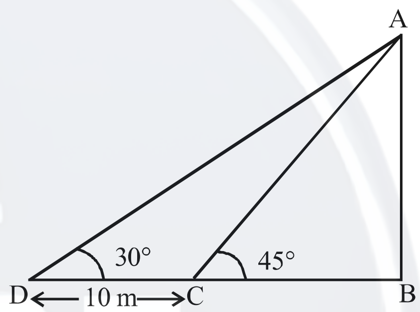
$\dfrac{1}{\sqrt{3}} = \dfrac{10}{EB} \Rightarrow EB = 10\sqrt{3}$m

Putting value of EB in (1)

$AB = \sqrt{3}\left(10\sqrt{3}\right) = 30$m

AC = AB + BC = 40 m

75. (a)



In ΔABC, $\tan 45° = \dfrac{AB}{BC}$

$1 = \dfrac{AB}{BC} \Rightarrow AB = BC$ ...(1)

In ΔABD, $\tan 30° = \dfrac{AB}{BD}$

$\dfrac{1}{\sqrt{3}} = \dfrac{AB}{BD}$

$BD = AB\sqrt{3}$

$DC + BC = AB\sqrt{3}$

$DC = AB\sqrt{3} - AB$ [from (1)]

$10 = AB\left(\sqrt{3} - 1\right)$

$AB = \dfrac{10}{\sqrt{3} - 1} \times \dfrac{\sqrt{3} + 1}{\sqrt{3} + 1} = 5\left(\sqrt{3} + 1\right)$m

76. (b) $\dfrac{\tan^2\theta}{1 + \tan^2\theta} + \dfrac{\cot^2\theta}{1 + \cot^2\theta}$

$= \dfrac{\tan^2\theta}{\sec^2\theta} + \dfrac{\cot^2\theta}{\operatorname{cosec}^2\theta}$

$= \sin^2\theta + \cos^2\theta = 1$