Answer & Solutions

Level-I

1. (a) When two are thrown then there are 6 × 6 exhaustive cases $\therefore n = 36$. Let A denote the event "total score of 7" when 2 dice are thrown then A = [(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)].
Thus there are 6 favourable cases.

$\therefore m = 6$ By definition $P(A) = \frac{m}{n}$

$\therefore P(A) = \frac{6}{36} = \frac{1}{6}$.

2. (a) $\because$ No of ways of drawing 2 white balls from 5 white balls = ${}^5C_2$.
Also, No of ways of drawing 2 other from remaining 7 balls = ${}^7C_2$
Total number of balls = 12

Hence, required probability $= \frac{{}^5C_2 \times {}^7C_2}{{}^{12}C_4} = \frac{14}{33}$

3. (b) Total no. of outcomes when two dice are thrown = $n(S)$ = 36 and the possible cases for the event that the sum of numbers on two dice is a prime number, are (1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 1), (5, 6), (6, 1), (6, 5)
Number of outcomes favouring the event = $n(A) = 15$

Required probability $= \frac{n(A)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

4. (c) $P(A) = \frac{1}{3}$, $P(\bar{A}) = \frac{2}{3}$

P (bird killed) = 1 – P (none of 3 shots hit)

$= 1 - \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{19}{27}$.

5. (b) Since, A and B are independent events
$\therefore A'$ and B' are also independent events

$\Rightarrow P(A' \cap B') = P(A').P(B')$

$= (0.4)(0.7) = 0.28$

$[\because P(A') = 1 - P(A),\ P(B') = 1 - P(B)]$

6. (d)

7. (d) Doublets occur when the numbers thrown are (1, 1), (2, 2), ..., (6, 6). Therefore the probability of a doublet occurring in single throw $= \frac{6}{36} = \frac{1}{6}$.

The probability of a doublet not occurring at all in three throws $= \left(\frac{5}{6}\right)^3 = \frac{125}{216}$.

Required probability $= 1 - \frac{125}{216} = \frac{91}{216}$.

8. (c) Required probability = 1/6.

9. (b) Total probable ways = 8
Favourable number of ways = HTH, THT

Hence required probability $= \frac{2}{8} = \frac{1}{4}$

10. (b) Any of the six numbers 1,2,3,4,5,6 may appear on the upper face. $\therefore n = 6$
Number of odd numbers = 3, since the odd numbers are 1, 3, 5
$\therefore m = 3$.
$\therefore$ The required probability

$= \frac{\text{number of favourable cases}}{\text{number of all cases}} = \frac{m}{n} = \frac{3}{6} = \frac{1}{2}$

11. (d) n = Number of all cases = 6
m = Number of favourable cases = 4 (since the numbers that appear are 3, 4, 5, 6)

$\therefore$ The required probability $= p = \frac{m}{n} = \frac{4}{6} = \frac{2}{3}$

12. (a) $S = (1, 2, 3, 4, 5, 6)$ $\therefore n(S) = 6$
Let A be the event that the die shows a multiple of 2.
$A = \{2, 4, 6\}$ $\therefore n(A) = 3$

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

13. (a) India win atleast three matches

$= {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_4\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^5 (16)$

$= \left(\frac{1}{2}\right)$

14. (d) Required probability

$= {}^5C_3\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2 + {}^5C_4\left(\frac{3}{4}\right)^4.\left(\frac{1}{4}\right) + {}^5C_5\left(\frac{3}{4}\right)^5 = \frac{459}{512}$

15. (b) Total no. of numbers divisible by 4 between 1 to 80
$80 = 4 + (n - 1)4$
$80 = 4n$
$\Rightarrow n = 20$

$\therefore$ Required probability $= \frac{{}^{20}C_2}{{}^{80}C_2} = \frac{19}{316}$

16. **(b)** Here $n(S) = 6^2 = 36$

Let E be the event "getting sum more than 7" *i.e.* sum of pair of dice = 8, 9, 10, 11, 12

$$i.e.,\ E = \begin{Bmatrix} (2,6), & (3,5), & (4,4), & (5,3), & (6,2), \\ (3,6), & (4,5), & (5,4), & (6,3), & \\ (4,6), & (5,5), & (6,4), & & \\ (5,6), & (6,5), & (6,6) & & \end{Bmatrix}$$

$\therefore\ n(E) = 15$

$\therefore$ Required prob $= \dfrac{n(E)}{n(S)} = \dfrac{15}{36} = \dfrac{5}{12}$

17. **(c)** Number of sample points on throwing two dice

$= 6 \times 6 = 36$

The possible outcomes are (1, 4), (2, 3), (3, 2), (4, 1)

The probability of obtaining a total score of 5 is

$= \dfrac{4}{6\times 6} = \dfrac{1}{9}$.

18. **(c)** Given digits are 1, 2, 3, 4, 5

Total no. of 2 digits numbers formed $= (5)^2 = 25$

Favourable cases are 12, 24, 32, 44, 52

No. of favourable cases = 5

$\therefore$ Required Probability $= \dfrac{5}{25} = \dfrac{1}{5}$

19. **(a)** Total no. of cases $= 6^3 = 216$

16 can appear on three dice in following ways

(6, 6, 4), (6, 5, 5), (6, 4, 6), (4, 6, 6), (5, 5, 6), (5, 6, 5).

$\therefore$ No. of favourable cases = 6

Hence, the required probability $= \dfrac{6}{6^3} = \dfrac{1}{36}$

20. **(b)** Total number of bolts = 600

Number of too large bolts = 20% of 600

$= \dfrac{20\times 600}{100} = 120$

Number of too small bolts = 10% of 600 = 60

Number of suitable bolts = 600 – 120 – 60 = 420

Thus required probability $= \dfrac{420}{600} = \dfrac{7}{10}$

21. **(c)** Total possible outcomes = 36

E = Event of getting sum 7

= {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}

F = Event of getting sum 11

= {(6,5), (5,6)}

Total no. of favourable cases = 6 + 2 = 8

Now required probability

$= \dfrac{\text{Total favourable cases}}{\text{Total outcomes}} = \dfrac{8}{36} = \dfrac{2}{9}$

22. **(a)** $n(S) = 100$

E = square numbers from 1 to 100.

= 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

$n(E) = 10$

$\therefore$ Required probability $= \dfrac{n(E)}{n(S)} = \dfrac{10}{100} = \dfrac{1}{10}$

23. **(a)** (AAAA), (LL), HBD

$$P = \frac{\frac{5!}{\frac{9!}{4!\,2!}}}{} = \frac{5!\times 4!\times 2!}{9!} = \frac{24\times 2}{9\times 8\times 7\times 6} = \frac{1}{63}$$

24. **(b)** The probability that Krishna will be alive 10 years hence, is $\dfrac{7}{15}$

So, probability that Krishna will be dead 10 years hence, the

$= 1 - \dfrac{7}{15} = \dfrac{8}{15}$

Also, probability that Hari will be alive 10 years hence is $\dfrac{7}{10}$

So, the probability that Hari will be dead 10 years hence, $= 1 - \dfrac{7}{10} = \dfrac{3}{10}$

So, the probability that both Krishna and Hari will be dead 10 years hence

$= \dfrac{8}{15} \times \dfrac{3}{10} = \dfrac{24}{150}$

25. **(a)** Total no. of arrangements of the letters of the word UNIVERSITY is $\dfrac{10!}{2!}$.

No. of arrangements when both I's are together = 9!

So. the no. of ways in which 2 I's do not together

$= \dfrac{10!}{2!} - 9!$

$\therefore$ Required probability

$$= \frac{\frac{10!}{2!} - 9!}{\frac{10!}{2!}} = \frac{10! - 9!\,2!}{10!}$$

26. **(a)** Total no. of players = 15

Total no. of batsmen = 8

Total no. of bowlers = 7

Total no. of players in the team = 11

$\therefore$ No. of ways to choose a team $= {}^{15}C_{11}$

$\therefore$ No. of way to choose 6 batsmen and 5 bowler

$= {}^{8}C_{6} \times {}^{7}C_{5}$

$\therefore$ Required Probability $= \dfrac{{}^{8}C_{6}\times {}^{7}C_{5}}{{}^{15}C_{11}}$

27. **(d)** Total number of numbers = 4! = 24
For odd nos. 1 or 3 has to be at unit's place
If 1 is at unit place, then total number of numbers
= 3! = 6
And if 3 is at units place, then total number of numbers
= 3! = 6
$\therefore$ Total number of odd number = 6 + 6 = 12

$\therefore$ Required probability $= \frac{12}{24} = \frac{1}{2}$

28. **(c)** Required Probability $= P(X).P(\bar{Y}) + P(\bar{X}).P(Y)$

$= \frac{60}{100} \times \frac{50}{100} + \frac{40}{100} \times \frac{50}{100} = \frac{1}{2}$

29. **(c)** $P(3 \cup 4) = P(c) + P(d) - P(3 \cap 4)$

$= \frac{8}{25} + \frac{6}{25} - \frac{2}{25} = \frac{12}{25}$

30. **(b)** In a leap year there are 366 days in which 52 weeks and two days. The combination of 2 days may be : Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun.

P (53 Fri) $= \frac{2}{7}$; P (53 Sat) $= \frac{2}{7}$

and P (53 Fri and 53 Sat) $= \frac{1}{7}$

$\therefore$ P (53 Fri or Sat) = P (53 Fri) + P (53 Sat) − P (53 Fri and Sat)

$= \frac{2}{7} + \frac{2}{7} - \frac{1}{7} = \frac{3}{7}$

31. **(d)** Clearly, $P(A \cup B \cup C) = 1$
$\Rightarrow$ $P(A) + P(B) + P(C) = 1$

$\Rightarrow$ $P(A) + \frac{1}{2}P(A) + \frac{1}{3}P(A) = 1$

$\Rightarrow$ $\frac{11}{6}P(A) = 1$

$\Rightarrow$ $P(A) = \frac{6}{11}$

32. **(d)** Now, $P(A' \cap B') = P(A' \cup B')$
$= 1 - P(A \cup B) = 1 - 0.8 = 0.2$
and $P(A' \cup B') = 1 - P(A \cap B) = 1 - 0.3 = 0.7$
But $P(A' \cup B') = P(A') + P(B') - P(A' \cap B')$
$\Rightarrow$ $0.7 = P(A') + P(B') - 0.2$
$\Rightarrow$ $P(A') + P(B') = 0.9$.

33. **(c)** Let E be the event of total of 12.
E = (2, 2, 2, 3, 3), (2, 2, 3, 3, 2), (2, 3, 3, 2, 2), (3, 3, 2, 2, 2), (3, 2, 3, 2, 2), (3, 2, 2, 3, 2), (3, 2, 2, 2, 3), (2, 3, 2, 3, 2), (2, 3, 2, 2, 3), (2, 2, 3, 2, 3)
$n(E) = 10$
Sample sapce contain total possibility $= 2^5 = 32$
Hence, $n(s) = 32$

So, $P(E) = \frac{n(E)}{n(S)} = \frac{10}{32} = \frac{5}{16}$

34. **(c)** Since, probabilities of failure for engines A, B and C $P(A)$, $P(B)$ and $P(C)$ are 0.03, 0.02 and 0.05 respectively.
The aircraft will crash only when all the three engine fail. So, probability that it crashes = $P(A)$. $P(B)$. $P(C)$
$= 0.03 \times 0.02 \times 0.05 = 0.00003$
Hence, the probability that the aircraft will not crash,
$= 1 - 0.00003 = 0.99997$

35. **(a)** Probability of passing in mathematics $= \frac{4}{9}$

Probability of passing in physics $= \frac{2}{5}$

Probability of failure in physics $= 1 - \frac{2}{5} = \frac{3}{5}$
Given that both the events are independent.

Required probability $= \frac{4}{9} \times \frac{3}{5} = \frac{4}{15}$

36. **(c)** Probability of getting a diamond, $P(D) = \frac{13}{52} = \frac{1}{4}$

and probability to king, $P(K) = \frac{4}{52} = \frac{1}{13}$

So, required probability = $P(D).P(K)$

$= \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$

37. **(c)** 16 tickets are sold and 4 prizes are awarded. A person buys 4 tickets, then required probability $= \frac{4}{16} = \frac{1}{4}$

38. **(b)** If both get one head then it is $\frac{1}{4} \times \frac{1}{4}$

and if both get two heads then it is $\frac{1}{2} \times \frac{1}{2}$

$\Rightarrow$ Prob (getting same number of heads) $= \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{16} + \frac{1}{4} = \frac{5}{16}$

39. **(c)** Let $P(A)$ be the probability that the race will be won by A and $P(B)$ be the probability that the race will be won by B.

$\therefore$ $P(A) = \frac{1}{5}$ and $P(B) = \frac{1}{6}$

$\therefore$ Probability that the race will be won by

A or $B = P(A) + P(B) = \frac{1}{5} + \frac{1}{6} = \frac{11}{30}$

40. **(b)** Required probability $= \frac{365}{365} \times \frac{1}{365} = \frac{1}{365}$.

41. **(c)** As we know $P(A \cup B) \le 1$

$\therefore\ P(A) + P(B) - P(A \cap B) \le 1$

$\Rightarrow 0.8 + 0.7 - P(A \cap B) \le 1$

$\Rightarrow P(A \cap B) \ge 1.5 - 1$

$\Rightarrow P(A \cap B) \ge 0.5$

Hence, the minimum value of $P(A \cap B)$ is 0.5.

42. **(c)** Possible samples are as follows

$\{HHH, HTH, HHT, THH, TTH, THT, HTT, TTT\}$

Let A be the event of getting one head.

Let B be the event of getting no head.

Favourable outcome for

$A = \{TTH, THT, HTT\}$

Favourable outcome for

$B = \{TTT\}$

Total no. of outcomes = 8

$\therefore\ P(A) = \frac{3}{8},\ P(B) = \frac{1}{8}$

$\therefore$ Required probability = Probability of getting one head + Probability of getting no head

$= P(A) + P(B) = \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

43. **(a)** No. of blue balls =2

No. of red balls = 7

Total no. of balls = 9

Required probability

= P (one ball is blue) + P (both ball is blue)

$= \frac{2}{9} \times \frac{7}{8} + \frac{2}{9} \times \frac{1}{8} = \frac{14}{72} + \frac{2}{72} = \frac{16}{72} = \frac{2}{9}$

44. **(c)** Given probability of guessing a correct answer $= \frac{x}{12}$

and probability of not guessing the correct answer $= \frac{2}{3}$

As we know

P (occurence of an event) + P (non-occurence of an event) = 1

$\therefore\ \frac{x}{12} + \frac{2}{3} = 1 \Rightarrow \frac{x+8}{12} = 1 \Rightarrow x = 12 - 8 = 4$

45. **(d)** Probability that only husband is selected

$= P(H)P(\overline{W}) = \frac{1}{7}\left(1 - \frac{1}{5}\right) = \frac{1}{7} \times \frac{4}{5} = \frac{4}{35}$

Probability that only wife is selected

$= P(\overline{H})P(W) = \left(1 - \frac{1}{7}\right)\left(\frac{1}{5}\right) = \frac{6}{7} \times \frac{1}{5} = \frac{6}{35}$

$\therefore$ Probability that only one of them is selected

$= \frac{4}{35} + \frac{6}{35} = \frac{10}{35} = \frac{2}{7}$

46. **(b)** The probability that the person hits the target = 0.3

$\therefore$ The probability that he does not hit the target in a trial = $1 - 0.3 = 0.7$

$\therefore$ The probability that he does not hit the target in any of the ten trials = $(0.7)^{10}$

$\therefore$ Probability that he hits the target

= Probability that at least one of the trials succeeds

$= 1 - (0.7)^{10}$.

47. **(c)** If six coins are tossed, then the total no. of outcomes $= (2)^6 = 64$

Now, probability of getting no tail $= \frac{1}{64}$

Probability of getting at least one tail

$= 1 - \frac{1}{64} = \frac{63}{64}$

48. **(d)** Total of seven can be obtained in the following ways

1, 1, 1, 4 in $\frac{4!}{3!} = 4$ ways

[there are four objects, three repeated]

Similarly,

1, 1,2, 3 in $\frac{4!}{2!} = 12$ ways

1, 2,2, 2 in $\frac{4!}{3!} = 4$ ways

Hence, required probability $= \frac{4+12+4}{6^4} = \frac{20}{6^4}$

[$\because$ Exhaustive no. of cases $= 6 \times 6 \times 6 \times 6 = 6^4$]

49. **(c)** The number of ways of getting the different number 1, 2, ..., 6 in six dice = 6 !.

Total number of ways $= 6^6$

Hence, required probability $= \frac{6!}{6^6}$

$= \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6}{6^6} = \frac{5}{324}$

50. **(a)** Here the number of favourable cases, consists of throwing 10,11 or 12 with the two dice. The number of ways in which a sum of 10 can be thrown are (4,6), (5,5), (6,4) i.e. 3 ways. The number of ways in which a total of 11 can be thrown are (5, 6), (6,5) i.e. 2 ways. The number of ways in which a total of 12 can be thrown in (6, 6) i.e. 1 way.

m = number of favourable cases = 3 + 2 + 1 = 6

n = Total number of cases = 6 × 6 = 36

$\therefore$ Probability $= p = \frac{m}{n} = \frac{6}{36} = \frac{1}{6}$

51. **(c)** Required probability

$= P\text{ (Diamond)} . P\text{ (King)} = \frac{13}{52}.\frac{4}{52} = \frac{1}{52}$

52. **(a)** The cards are of four colours and the number of cards of given description is 24.

The probability $= \frac{24}{52}.\frac{23}{51} = \frac{46}{221}$.

53. **(b)** Since A and B are independent

$\therefore\ P(A \cap B) = P(A).P(B)$

and $P(A/B) = P(A)$

Thus, $P(A/B) = \frac{1}{2}$

Hence, option (b) is not true.

54. **(c)** The probability that a man will not live 10 more years = 3/4 and the probability that his wife will not live 10 more years = 2/3. Then the probability that neither will be alive in 10 years = 3/4 × 2/3 = 1/2

55. **(b)** Total number of possibilities = 25 × 25

Favourable cases for their winning = 25

$\therefore\ P\text{ (they win a prize)} = \frac{25}{25 \times 25} = \frac{1}{25}$

$\therefore\ P\text{ (they will not win a prize)} = 1 - \frac{1}{25}$

56. **(d)** Here $P(A) = 0.4$ and $P(\bar{A}) = 0.6$

Probability that A does not happen at all = $(0.6)^3$

Thus required probability = $1 - (0.6)^3 = 0.784$

57. **(d)** As there are four jacks and four aces, the number of favourable cases = 8

$\therefore$ The required probability $p = \frac{8}{52} = \frac{2}{13}$

58. **(a)** The favourable cases are (1, 3), (2, 4), (3,5), (4,6) and (1, 4), (2, 5), (3, 6) and their reversed cases like (3, 1)......

Total number of favourable cases = 2 × 7

$\therefore\ p = \frac{14}{36} = \frac{7}{18}$

59. **(a)** The first card can be one of the 4 colours, the second can be one of the three and the third can be one of the two. The required probability is therefore

$4 \times \frac{13}{52} \times 3 \times \frac{13}{51} \times 2 \times \frac{13}{50} = \frac{169}{425}$.

60. **(c)** Total number of ways in which 4 persons can be selected out of 3 + 2 + 4 = 9 persons = ${}^9C_4 = 126$

Number of ways in which a selection of 4 contains exactly 2 children = ${}^4C_2 \times {}^5C_2 = 60$

$\therefore$ reqd. prob. $= \frac{60}{126} = \frac{10}{21}$

61. **(b)** $P(A) = 1/4,\ P(A/B) = \frac{1}{2},\ P(B/A) = 2/3$

By conditional probability,

$P(A \cap B) = P(A)\,P(B/A) = P(B)P(A/B)$

$\Rightarrow \frac{1}{4} \times \frac{2}{3} = P(B) \times \frac{1}{2} \Rightarrow P(B) = \frac{1}{3}$

62. **(c)** Probability of getting a head on tossing a coin $(P_1) = \frac{1}{2}$.

Probability of getting a six on rolling a dice $(P_2) = \frac{1}{6}$.

These two events are independent.

So the probability that the coin shows the head and the dice shows 6 is given by

$P = P_1 \times P_2 = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$.