Answer & Solutions

Level-I

1. **(d)** Equations in options (a) and (c) are not quadratic equations as in (a) max. power of x is fractional and in (c), it is not 2 in any of the terms.

For option (b), $(x-1)(x+4) = x^2+1$

or $\quad x^2+4x-x-4 = x^2+1$

or $\quad 3x-5=0$

which is not a quadratic equations but a linear.

For option (d), $(2x+1)(3x-4) = 2x^2+3$

or $\quad 6x^2-8x+3x-4 = 2x^2+3$

or $\quad 4x^2-5x-7=0$

which is clearly a quadratic equation.

2. **(a)** $x-\dfrac{1}{x} = 1\dfrac{1}{2} \Rightarrow \dfrac{x^2-1}{x} = \dfrac{3}{2}$

$\Rightarrow \quad 2(x^2-1) = 3x \Rightarrow 2x^2-2 = 3x$

$\Rightarrow \quad 2x^2-3x-2=0$

$\Rightarrow \quad 2x^2-4x+x-2=0$

$\Rightarrow \quad 2x(x-2)+1(x-2)=0$

$\Rightarrow \quad (2x+1)(x-2)=0$

Either $2x+1=0$ or $x-2=0$

$\Rightarrow \quad 2x=-1$ or $x=2$

$\Rightarrow \quad x=\dfrac{-1}{2}$ or $x=2$

$\therefore \quad x=\dfrac{-1}{2}$, 2 are solutions.

3. **(c)** $2x^2-7xy+3y^2=0$

$$2\left(\frac{x}{y}\right)^2-7\left(\frac{x}{y}\right)+3=0$$

$$\frac{x}{y}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{7\pm\sqrt{49-24}}{2\times 2}=\frac{7\pm 5}{4}=3,\frac{1}{2}$$

$\Rightarrow \quad \dfrac{x}{y}=\dfrac{3}{1}$ or $\dfrac{x}{y}=\dfrac{1}{2}$

4. **(b)** Let the son's age be x years.

So, father's age $= 5x-4$ years.

$\therefore \quad x(5x-4)=288$

$\Rightarrow \quad 5x^2-4x-288=0 \Rightarrow 5x^2-40x+36x-288=0$

$\Rightarrow \quad 5x(x-8)+36(x-8)=0$

$\Rightarrow \quad (5x+36)(x-8)=0$

Either $x-8=0$ or $5x+36=0 \Rightarrow x=8$ or $x=\dfrac{-36}{5}$

x cannot be negative; therefore, $x = 8$ is the solution.

$\therefore$ Son's age = 8 years and Father's age = $5x-4$ = 36 years.

5. **(a)** Let the number be x.

Then, $x+\dfrac{1}{x}=\dfrac{13}{6} \Rightarrow \dfrac{x^2+1}{x}=\dfrac{13}{6} \Rightarrow 6x^2-13x+6=0$

$\Rightarrow 6x^2-9x-4x+6=0 \Rightarrow (3x-2)(2x-3)=0$

$\Rightarrow x=\dfrac{2}{3}$ or $x=\dfrac{3}{2}$.

Hence, the required number is $\dfrac{2}{3}$ or $\dfrac{3}{2}$.

6. **(b)** $\sqrt{6+\sqrt{6+\sqrt{6+\ldots\ldots\ldots\ldots}}}$

$\sqrt{6+x}=x$

$6+x=x^2$

$x^2-x-6=0$

$x^2-3x+2x-6=0$

$(x-3)(x+2)=0$

$x=3$

7. **(b)** $x^2+2=2x \Rightarrow x^2-2x+2=0$

$$\begin{array}{l}
x^2-2x+2\,)\,x^4-x^3+x^2+2\,(\,x^2+x+1\\
\qquad\qquad\quad x^4-2x^3+2x^2\\
\qquad\qquad\quad -\quad +\quad -\\
\hline
\qquad\qquad\quad x^3-x^2+2\\
\qquad\qquad\quad x^3-2x^2+2x\\
\qquad\qquad\quad -\quad +\quad -\\
\hline
\qquad\qquad\quad x^2-2x+2\\
\qquad\qquad\quad x^2-2x+2\\
\hline
\qquad\qquad\qquad 0
\end{array}$$

$\therefore \quad x^4-x^3+x^2+2$

$\quad = (x^2-2x+2)(x^2+x+1)=0$

8. **(d)** $x^2 \ge 0$

$\therefore$ Minimum value

$=0+\dfrac{1}{1}-3=-2$

9. **(d)** Given $x^2+kx-8=0$

Let a and b be the roots of given equation and $b=a^2$ (given)

Sum of roots $=a+b=-k=a+a^2$ .....(1)

Product of roots $=ab=-8=a^3 \Rightarrow a=-2$

Using $a=-2$ in (1), $-k=-2+4=2$ or $k=-2$

10. (a) $7x_2 - 4x_1 = 47$
$x_1 + x_2 = 2$
Solving $11x_2 = 55$
$x_2 = 5$ & $x_1 = -3$
$\therefore c = -15$

11. (c) Since, the roots of the equation $(k+1)x^2 - 2(k-1)x + 1 = 0$ are real and equal.
$\therefore \{-2(k-1)\}^2 - 4(k+1) = 0 \quad (\because b^2 - 4ac = 0)$
$\Rightarrow 4(k^2 - 2k + 1) - 4(k+1) = 0$
$\Rightarrow k^2 - 2k + 1 - 1 = 0$
$\Rightarrow k^2 - 3k = 0$
$\Rightarrow k = 0, k = 3$

12. (b) Since roots of the given equation are equal.
$\therefore D = 0$
On solving we get $b^2 = ac$

13. (b) Use $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

14. (a)

15. (a) Let $x = \sqrt{5\sqrt{5\sqrt{5 \ldots \infty}}}$
$\Rightarrow x^2 = 5x \Rightarrow x = 0, 5$

16. (d) $\frac{1}{r^2} + \frac{1}{s^2} = \frac{s^2 + r^2}{(rs)^2} = \frac{(s+r)^2 - 2sr}{(rs)^2} = \frac{p^2 - 2q}{q^2}$

17. (b) Clearly, the given equation is valid if $x - 3 \neq 0$ and $2x + 3 \neq 0$ i.e.,. when $x \neq \frac{-3}{2}, 3$

Now, $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$
$\Rightarrow 2x(2x+3) + (x-3) + 3x + 9 = 0$
[Multiplying throughout by $(x-3)(2x+3)$]
$\Rightarrow 4x^2 + 6x + x - 3 + 3x + 9 = 0$
$\Rightarrow 4x^2 + 10x + 6 = 0$
$\Rightarrow 2x^2 + 5x + 3 = 0$
$\Rightarrow 2x^2 + 2x + 3x + 3 = 0$
$\Rightarrow 2x(x+1) + 3(x+1) = 0$
$\Rightarrow (2x+3)(x+1) = 0$
$\Rightarrow x + 1 = 0 \Rightarrow x = -1 \; [\because 2x + 3 \neq 0]$
Hence, $x = -1$ is the only solution of the given equation.

18. (b) Since the roots of $x^2 - kx + 4 = 0$ are non-real.
$\therefore$ Disc., $(-k^2) - 4 < 0 \Rightarrow k^2 - 4 < 0$
$\Rightarrow k^2 < 4 \Rightarrow |k| < 2 \Rightarrow -2 < k < 2$

19. (b)

20. (d) Square both sides, we shall get $x = -4, \frac{1}{2}$. But both of them do not satisfy the given equation.

21. (b) $p + q = 18$ ...(1)

and $\frac{1}{p} + \frac{1}{q} = \frac{1}{4}$ ...(2) (Given)

i.e $\frac{p+q}{pq} = \frac{1}{4} \Rightarrow \frac{18}{pq} = \frac{1}{4}$
$\Rightarrow pq = 72$ ...(3)
From (1) and (3), $p(18 - p) = 72$
$\Rightarrow p^2 - 18p + 72 = 0 \Rightarrow (p-6)(p-12) = 0$
$\Rightarrow p = 6, 12$ when $p = 6, q = 12$; when $p = 12, q = 6$
Hence the numbers are 12, 6.

22. (b) Let the roots be α and $\alpha + 2$.
Then $\alpha + \alpha + 2 = b \Rightarrow \alpha = (b-2)/2$ (1)
and $\alpha(\alpha+2) = c \Rightarrow \alpha^2 + 2\alpha = c$ (2)
Putting the value of α from (1) in (2).
$((b-2)/2)^2 + 2(b-2)/2) = c$
$\Rightarrow (b^2 + 4 - 4b)/4 + b - 2 = c$
$\Rightarrow b^2 + 4 - 8 = 4c$
$\Rightarrow b^2 = 4c + 4$

23. (c) Let the number be x. Then,

$x + \frac{1}{x} = \frac{26}{5}$

$\Rightarrow \frac{x^2+1}{x} = \frac{26}{5}$
$\Rightarrow 5x^2 - 26x + 5 = -0$
$\Rightarrow (x-5)(5x-1) = 0$
$\Rightarrow x = 5$ or $\frac{1}{5}$

24. (d)

25. (d) $x + \sqrt{x-2} = 4$
$\sqrt{x-2} = 4 - x$
Squaring on the both sides
$x - 2 = 16 + x^2 - 8x$
$x^2 - 9x + 18 = 0$
$(x-6)(x-3) = 0$
$x = 6$ or 3
But by checking, only $x = 3$ satisfies the equation.

26. (b) $\sqrt{x+10} - \frac{6}{\sqrt{x+10}} = 5$
$x + 10 - 6 = 5\sqrt{x+10}$
$x + 4 = 5\sqrt{x+10}$
Squaring on both sides,
$x^2 + 8x + 16 = 25x + 250$
$x^2 - 17x - 234 = 0$
$x^2 - 26x + 9x - 234 = 0$
$x(x-26) + 9(x-26) = 0$
$(x-26)(x+9) = 0$
$x = 26$ (or) -9
Here $x = -9$ is not satisfying. So it is extraneous.

27. **(d)** $\log_{10}(x^2-3x+6)=1$

$x^2-3x+6=10^1$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

$x=4$ or -1

28. **(c)** $2\sqrt{x}+\frac{2}{\sqrt{x}}=5$

$2x+2=5\sqrt{x}$

$\Rightarrow 4x^2+8x+4=25x$

$\Rightarrow 4x^2-17x+4=0$

29. **(b)** Let α and β are the roots

$\alpha+\beta=6$

$\alpha-\beta=8$

$2\alpha=14$

$\alpha=7$

$\beta=-1$

$\alpha+\beta=6, \alpha\beta=-7$

The quadratic equation is $x^2-6x-7=0$

30. **(a)** $b^2-4ac=(2\sqrt{3})^2-4(1)(3)=0$. So the roots are real and equal.

31. **(c)** Since roots are reciprocal,

product of the roots $=1 \Rightarrow \frac{c}{a}=1$

$\Rightarrow c=a$.

32. **(a)** $\frac{b}{x-a}=\frac{x+a}{b}$

$x^2-a^2=b^2$

$x^2=b^2+a^2$

$x=\pm\sqrt{a^2+b^2}$

33. **(d)** Solve using options. It can be seen that $b=0$ and $c=0$ the condition is satisfied. It is also satisfied at $b=1$ and $c=-2$.

34. **(d)** $4\sqrt{3}x^2+5x-2\sqrt{3}$

$=4\sqrt{3}x^2+8x-3x-2\sqrt{3}$

$=4x(\sqrt{3}x+2)-\sqrt{3}(\sqrt{3}x+2)$

$=\left(4x-\sqrt{3}\right)\left(\sqrt{3}x+2\right)$

35. **(c)** $\frac{3x^2-4x+3}{x^2-x+1}=\frac{\frac{3x^2}{x}-\frac{4x}{x}+\frac{3}{x}}{\frac{x^2}{x}-\frac{x}{x}+\frac{1}{x}}$

$$\frac{3\left(x+\frac{1}{x}\right)-4}{\left(x+\frac{1}{x}\right)-1}=\frac{3\times3-4}{3-1}=\frac{5}{2}$$

36. **(c)** $x^4-2x^2y^2+y^4=(x^2-y^2)^2=[(x+y)(x-y)]^2$

$=\left(2p\times\frac{2}{p}\right)^2=16$

37. **(d)** We have, $x=3+2\sqrt{2}$

$\frac{1}{x}=\frac{1}{3+2\sqrt{2}}\times\frac{3-2\sqrt{2}}{3-2\sqrt{2}}=3-2\sqrt{2}$

$x+\frac{1}{x}=6$

$\frac{x^6+x^4+x^2+1}{x^3}=x^3+x+\frac{1}{x}+\frac{1}{x^3}$

$=\left(x^3+\frac{1}{x^3}\right)+\left(x+\frac{1}{x}\right)$

$=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)+\left(x+\frac{1}{x}\right)$

$=\left(x+\frac{1}{x}\right)\left[\left(x+\frac{1}{x}\right)^2-3\right]+\left(x+\frac{1}{x}\right)$

$=6[6^2-3]+6=198+6=204$

38. **(d)** Let x be the price of one capsule

y be the total number of capsule.

$xy=216$ ...(1)

$(x-10)(y+15)=216$ ...(2)

From eqs (1) and (2)

$\left(\frac{216}{y}-10\right)(y+15)=216$

$(216-10y)(y+15)=216y$

$216y+216\times15-10y^2-150y=216y$

$216y+3240-10y^2-150y=216y$

$-10y^2-150y+3240=0$

$y^2+15y-324=0$

$y=12,-27$

Number of capsules cannot be negative.