PRACTICE SET-3

1. The smallest of $\sqrt{8}+\sqrt{5}$. $\sqrt{7}+\sqrt{6}$. $\sqrt{10}+\sqrt{3}$ and $\sqrt{11}+\sqrt{2}$ and is :
   (a) $\sqrt{8}+\sqrt{5}$ (b) $\sqrt{7}+\sqrt{6}$
   (c) $\sqrt{10}+\sqrt{3}$ (d) $\sqrt{11}+\sqrt{2}$
2. Find the highest power of 5 in 100!.
   (a) 19 (b) 22
   (c) 25 (d) None of these
3. A contract is to be completed in 46 days and 117 men were set to work, each working 8 hours a day. After 33 days, 4/7 of the work is completed. How many additional men may be employed so that the work may be completed in time, each man now working 9 hours a day ?
   (a) 80 (b) 81
   (c) 82 (d) 83
4. A water tank has three taps A, B and C. A fills four buckets in 24 minutes, B fills 8 buckets in 1 hour and C fills 2 buckets in 20 minutes. If all the taps are opened together a full tank is emptied in 2 hours. If a bucket can hold 5 litres of water, what is the capacity of the tank?
   (a) 120 litres (b) 240 litres
   (c) 180 litres (d) 60 litres
5. A can do 50% more work as B can do in the same time. B alone can do a piece of work in 20 hours. A, with the help of B, can finish the same work in how many hours ?
   (a) 12 (b) 8
   (c) $13\frac{1}{3}$ (d) $5\frac{1}{2}$
6. Base of a right pyramid is a square. length of diagonal of the base is $24\sqrt{2}$ is the volume of the pyramid is 1728 cu.m. its height is
   (a) 7 m (b) 8 m
   (c) 9 m (d) 10 m
7. Which one set of letters when sequentially placed at the gaps in the given letter series shall complete it ?
   __ ab__ b __ aba __ __ abab
   (a) a ab aa (b) bb aa b
   (c) ab aa b (d) a aa ba

DIRECTIONS (Qs. 8) : *In this question, a series is given, with one term missing. Choose the correct alternative from the given ones that will complete the series.*

8. 121, 144, 289, 324, 529, 576, $\frac{?}{}$
   (a) 961 (b) 841
   (c) 900 (d) 729
9. A trader marked his goods at 20% above the cost price. He sold half the stock at the marked price, one quarter at a discount of 20% on the marked price and the rest at a discount of 40% on the marked price. His total gain is
   (a) 2% (b) 4.5%
   (c) 13.5% (d) 15%
10. A trader wants 10% profit on the selling price of a product whereas his expenses amount to 15% on sales. What should be his rate of mark up on an article costing ₹ 9?
    (a) 20% (b) $66\frac{2}{3}\%$
    (c) 30% (d) $\frac{100}{3}\%$
11. A manufacturer sells goods to an agent at a profit of 20%. The agent's wholesale price to a shopkeeper is at a profit of 10% and the shopkeeper retails his goods at a profit of 12%. Find the retailer's price of an article which had cost the manufacturer ₹ 25.
    (a) ₹ 37 (b) ₹ 40
    (c) ₹ 44 (d) ₹ 46
12. A cistern cotains 50 litres of water, 5 liters of water is taken out of it and replaced by wine. The process is repeated again. Find the proportion of wine and water in the resulting mixture.
    (a) 1 : 4 (b) 41 : 50
    (c) 19 : 81 (d) 81 : 19
13. A jar full of whisky contains 40% alcohol. A part of this whisky is replaced by another containing 19% alcohol and now the percentage of alcohol was found to be 26%. The quantity of whisky replaced is:
    (a) $\frac{1}{3}$ (b) $\frac{2}{3}$
    (c) $\frac{2}{5}$ (d) $\frac{3}{5}$
14. The average age of boys in class is 16.66, while the average age of girls is 18.75. Thus the average age of all the 40 students of the class is 17.5. If the difference between the no. of boys and girls is 8, then the no. of girls in the class is:
    (a) 12 (b) 16
    (c) 18 (d) Data insufficient
15. In a school, the average age of students is 6 years, and the average age of 12 teachers is 40 years. If the average age of the combined group of all the teachers and student is 7 years, then the number of student is :
    (a) 396 (b) 400
    (c) 408 (d) 416

16. An article is listed at ₹ 65. A customer bought this article for ₹ 56.16 and got two successive discounts of which the first one is 10%. The other rate of discount of this scheme that was allowed by the shopkeeper was
(a) 3% (b) 4%
(c) 6% (d) 2%

17. A's income is $6\frac{1}{4}\%$ more than B's. How much % is B's less than A's ?
(a) 5.89% (b) 4.78%
(c) 2.39% (d) None of these

18. A train leaves station X at 5 a.m. and reaches station Y at 9 a.m. Another train leaves station Y at 7 a.m. and reaches station X at 10: 30 a.m. At what time do the two trains cross each other ?
(a) 7 : 36 am (b) 7 : 56 am
(c) 8 : 36 am (d) 8 : 56 am

19. An athlete runs to and fro between points A and B at a speed of 10 km/h. A second athlete simultaneously runs from point B to A and back at a speed of 15 km/h. If they cross each other 12 min after the start, after how much time will they cross each other?
(a) 18 min (b) 24 min
(c) 36 min (d) 48 min

20. If $x^{\frac{1}{3}} + y^{\frac{1}{3}} = z^{\frac{1}{3}}$, then $(x + y - z)^3 + 27\,xyz$ equals :
(a) –1 (b) 1
(c) 0 (d) 27

21. Karnal took ₹ 6800 as a loan which along with interest is to be repaid in two equal annual instalments. If the rate of Interest is $12\frac{1}{2}\%$ compounded annually, then the value of each instalment is
(a) ₹ 8100 (b) ₹ 4150
(c) ₹ 4050 (d) ₹ 4000

22. If $a+\frac{1}{b} = 1$ and $b+\frac{1}{c} = 1$, then $c+\frac{1}{a}$ is equal to
(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) 2

23. The area of the triangle formed by the line $5x - 3y + 15 = 0$ with coordinate axes is
(a) 15 sq. units (b) 5 sq. units
(c) 8 sq. units (d) $\frac{15}{2}$ sq. units

24. The value of k for which the lines $x + 2y - 9 = 0$ and $kx + 4y + 5 = 0$ are parallel, is
(a) $k = 2$ (b) $k = 1$
(c) $k = -1$ (d) $k = -2$

25. If sec θ + tan θ = = p, then cos θ is
(a) $\frac{p^2+1}{p^2-1}$ (b) $\frac{p^2-1}{(p^2+1)^2}$
(c) $\frac{2p}{p^2+1}$ (d) $\frac{4p^2}{(p^2+1)^2}$

26. If tan A + sin A = p and tan A – sin A = q, them
(a) $p^2 + q^2 = 4\sqrt{pq}$ (b) $p^2 - q^2 = 4\sqrt{pq}$
(c) $p^2 - q^2 = \sqrt{pq}$ (b) $p^2 - q^2 = 2\sqrt{pq}$

27. The area of circle whose radius is 6 cm is trisected by two concentric circles. The radius of the smallest circle is
(a) $2\sqrt{3}$ cm (b) $2\sqrt{6}$ cm
(c) 2 cm (d) 3 cm

28. PQ and RS are two parallel chords of a circle with centre C such that PQ = 8 cm RS = 16 cm. If the chords are on the same side of the centre and the distance between them is 4 cm, then the radius of the circle is
(a) $3\sqrt{2}$ cm (b) $3\sqrt{5}$
(c) $4\sqrt{5}$ cm (d) $5\sqrt{5}$

29. If $ABCD$ is a parallelogram and E, F are the centroids of ΔABC and $\Delta\ BCD$ respectively, then EF is equal to
(a) AE (b) BE
(c) CE (d) DE

30. If the angle of elevation of the Sun changes from 30° to 45°, the length of the shadow of a pillar decreases by 20 metres. The height of the pillar is
(a) 20 $(\sqrt{3}-1)$ m (b) 20 $(\sqrt{3}+1)$ m
(c) 10 $(\sqrt{3}-1)$ (d) 10 $(\sqrt{3}+1)$ m

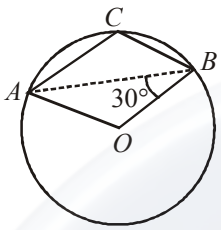
31. If cosec 39° = x, the value of $\frac{1}{\text{cosec}^2 51°}\ \sin^2 39° + \tan^2 51° - \frac{1}{\sin^2 51° \sec^2 39°}$ is
(a) $\sqrt{x^2-1}$ (b) $\sqrt{1-x^2}$
(c) $x^2 - 1$ (d) $1 - x^2$

32. If 3 cot θ = 4, then $\left(\frac{5\sin\theta+3\cos\theta}{5\sin\theta-3\cos\theta}\right)$ is equal to
(a) $\frac{1}{9}$ (b) $\frac{1}{3}$
(c) 3 (d) 9

33. If $\tan x + \cot x = 3$, then $\sec^2 + \text{cosec}^2 x$ is equal to
(a) 3 (b) 9
(c) 12 (d) 15

34. The greatest value of $\sin\theta + \cos\theta$ is equal to
(a) 0 (b) 1
(c) 2 (d) $\sqrt{2}$

35. In the following figure, O is the centre of the circle and $\angle ABO = 30°$, find $\angle ACB$.



(a) 60° (b) 120°
(c) 75° (c) 90°

36. A right triangle with sides 3 cm, 4 cm and 5 cm is rotated about the side of 3 cm to form a cone. The volume of the cone so formed is
(a) $16\pi\ \text{cm}^3$ (b) $12\pi\ \text{cm}^3$
(c) $15\pi\ \text{cm}^3$ (d) $20\pi\ \text{cm}^3$

37. If $x = 3 + 2\sqrt{2}$ then the value of $\sqrt{x} - \frac{1}{\sqrt{x}}$ is
(a) 1 (b) $2\sqrt{2}$
(c) 2 (d) $3\sqrt{3}$

38. If $a = 0.25$ $b = -0.05$, $c = 0.5$, then the value of $\frac{a^2 - b^2 - c^2 - 2bc}{a^2 + b^2 - 2ab - c^2}$ is
(a) $\frac{7}{8}$ (b) $\frac{14}{17}$
(c) 1 (d) $\frac{25}{16}$

39. If $x^2 - 3x + 1 = 0$, then the value of $x + \frac{1}{x}$ is
(a) 0 (b) 1
(c) 2 (d) 3

40. A chord of length 16 cm is drawn in a circle of radius 10 cm. The distance of the chord from the centre of the circle is
(a) 6 cm (b) 9 cm
(c) 12 cm (d) 8 cm

41. Each interior angle of a regular polygon is 18° more than eight times an exterior angle. The number of sides of the polygon is
(a) 10 (b) 15
(c) 20 (d) 25

42. The ratio of the outer and the inner perimeter of a circular path is 23 : 22. If the path is 5 m wide, the diameter of the inner circle is $\left(\text{use } \pi = \frac{22}{7}\right)$
(a) 110 m (b) 55 m
(c) 220 m (d) 230 m

43. A cistern has two pipes. One can fill it with water in 8 hours and other can empty it in 5 hours. In how many hours will the cistern be emptied if both the pipes are opened together when $\frac{3}{4}$ of the cistern is already full of water?
(a) $13\frac{1}{3}$ hours (b) 10 hours
(c) 6 hours (d) $3\frac{1}{3}$ hours

DIRECTIONS (Qs. 44-46) : *Study the following table carefully and answer the questions given below.*

Number of Students of Different Classes of a School Playing Difference Games

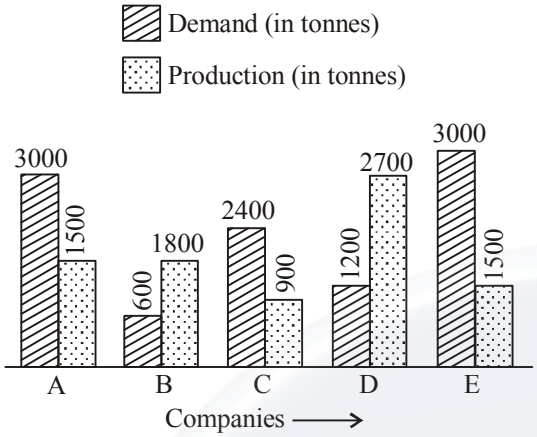
	XII	XI	X	IX	VIII	VII	VI
Chess	11	12	5	4	2	2	1
Cricket	38	40	12	17	25	18	20
Basket Ball	11	9	7	6	0	0	0
Table Tennis	9	9	21	19	11	9	0
Football	40	27	18	19	12	16	14
Carrom	16	15	8	19	12	16	14
Tennis	8	9	11	5	6	0	0
Badminton	47	39	33	21	19	0	0

44. Approximately what per cent of VIII class students play Cricket out of the total student playing Cricket?
(a) 13 (b) 4
(c) 25 (d) 15

45. What is the respective ratio of XI and X class students playing Football?
(a) 1 : 2 (b) 2 : 5
(c) 2 : 3 (d) 3 : 2

46. Which game is the most popular?
(a) Badminton (b) Cricket
(c) Carrom (d) Table Tennis

DIRECTIONS (Qs. 47-50) : *The following graph shows the demand and production of cotton by 5 companies A, B, C, D and E. Study the graph and answer the questions.*



47. What is the ratio of companies having more demand than production to those having more production than demand?
(a) 2 : 3 (b) 4 : 1
(c) 3 : 2 (d) 1 : 4

48. What is the difference (in tonnes) between average demand and average production of the five companies taken together?
(a) 320 (b) 420
(c) 2100 (d) 1050

49. The production of company D is how many times that of the production of the company A?
(a) 1.8 (b) 1.5
(c) 0.5 (d) 0.4

50. The demand for company B is what percent of the demand for comapany C?
(a) 1.5 (b) 2.5
(c) 25 (d) 30

SOLUTIONS

1. (d) Here, $\left(\sqrt{8}+\sqrt{5}\right)^2 = \left(\sqrt{8}\right)^2 + \left(\sqrt{5}\right)^2 + 2\times\sqrt{8}\times\sqrt{5}$

$= 8 + 5 + 2 \times \sqrt{8\times5} = 13 \quad 2\sqrt{40}$

Similarly,

$\sqrt{7} \quad \sqrt{6}^{\,2} \quad 7 \quad 6 \quad 2\times\sqrt{7\times6} = 13 \quad 2\sqrt{42},$

$\left(\sqrt{10}+\sqrt{3}\right)^2 = 10 \quad 3 \quad 2\times\sqrt{10\times3}$

$= 13 + 2\sqrt{30}, \quad \sqrt{11} \quad \sqrt{2}^{\,2}$

$= 11 + 2 + 2\sqrt{11\times2}$

$= 13 + 2\sqrt{22}$

Clearly, $13 + 2\sqrt{22}$ is the smallest among these.

$\therefore \quad \sqrt{11}+\sqrt{2}$ is the smallest.

2. (d) Divide 100 successively by 5 and keep on writing the quotient and then find the summation of all the quotient this summation will give us the highest power of 5 in 100!.

	100
5	20
5	4

Sum of all the quotient is 20 + 4 = 24, hence highest power of 5 in 100! *Is* 24.

Alternately

Required highest power of 5 is

$\left[\frac{100}{5^1}\right] \left[\frac{100}{5^2}\right] = 20 + 4 = 24$ hence highest power of 5 in 100! *Is* 24.

3. (b) Let x additional men employed.

117 men were supposed to finish the whole work in $46 \times 8 = 368$ hours.

But 117 men completed $\frac{4}{7}$ of the work in $33 \times 8 = 264$ hours

$\therefore$ 117 men could complete the work in 462 hours.

Now $(117 + x)$ men are supposed to do $\frac{3}{7}$ of the work, working 9 hours a day, in $13 \times 9 = 117$ hours, so as to finish the work in time.

i.e., $(117 + x)$ men are supposed to complete the whole work in $117 \times \frac{7}{3} = 273$ hours.

$\therefore \quad (117+x)\times 273 = 117 \times 462$

$\Rightarrow \quad (117+x)\times 7 = 3 \times 462$

$\Rightarrow \quad x + 117 = 3 \times 66 = 198 \Rightarrow x = 81$

$\therefore$ Required number of additional men to finish the work in time = 81.

4. (b) Tap A fills 4 buckets $(4 \times 5 = 20$ litres) in 24 min.

In 1 hour tap A fills $\frac{20}{24}\times 60 = 50$ litres

In 1 hour tap B fills $= 8\times5 = 40$ litres

In 1 hour tap C fills $\frac{2\times5}{20}\times 60 = 30$ litres

If they open together they would fill

$50 + 40 + 30 = 120$ litres in one hour

but full tank is emptied in 2 hours

So, tank capacity would be $120 \times 2 = 240$ litres.

5. (b) B alone can do a work in 20 hours.

$\therefore$ A alone can do $\frac{3}{2}$ of the work in 20 hours.

i.e., A alone can do the same work in $\frac{40}{3}$ hours

$\therefore$ (A + B)'s one hour's work $\frac{3}{40} \quad \frac{1}{20} \quad \frac{5}{40} \quad \frac{1}{8}$

$\Rightarrow$ A and B together can finish the whole work in 8 hours.

6. (c) Area of base $= \frac{1}{2}\times$ (diagonal)^2

$= \frac{1}{2}\times 24\sqrt{2}\times 24\sqrt{2} = 576$ sq. metre.

$\therefore$ Volume of pyramid $= \frac{1}{3}\times$ height $\times$ area of base

$\Rightarrow \quad 1728 = \frac{1}{3}\times h \times 576$

$\Rightarrow h = \frac{1728\times3}{576} = 9$ metre

7. (d) $\boxed{a} \quad a \quad b \quad \boxed{a} \quad b \;/\; \boxed{a} \quad a \quad b \quad a$

$\boxed{b}/\boxed{a} \quad a \quad b \quad a \quad b$

8. (b) $11^2, 12^2, 17^2, 18^2, 23^2, 24^2, 29^2, 30^2$

9. (a) Let C.P. of whole stock = ₹ 100.

Then, marked price of whole stock = ₹ 120.

M.P. of $\frac{1}{2}$ stock = ₹ 60, M.P. of $\frac{1}{4}$ stock = ₹ 30.

$\therefore$ Total S.P. = ₹ [60 + (80% of 30) + (60% of 30)]
= ₹ (60 + 24 + 18) = ₹ 102
Hence, gain% = (102 – 100)% = 2%.

10. (d) Let the SP of the article be ₹ x
Expenses = 15% of x = ₹ 0.15x
Profit = 10% of x = ₹ 0.10x
CP = ₹ 9 (given)
Therefore, $9 + 0.15x + 0.1x = x \Rightarrow x = 12$

$\therefore$ % increase for marked price $= \frac{12-9}{9} \times 100$

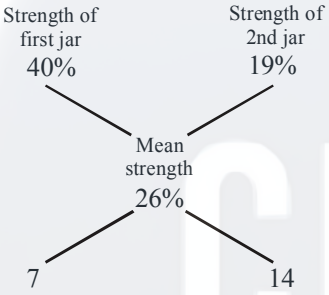
$= \frac{100}{3}\%$

11. (a) Retailer's price = 112 % of 110% of (120 % of 25)

$= \frac{112}{100} \times \frac{110}{100} \times \frac{120}{100} \times 25 = ₹\ 36.96 \approx ₹\ 37$

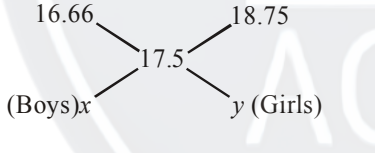
12. (c) Amount of water left = 50 × 9/10 × 9/10 = 40.5 litres. Hence, wine = 9.5 litres. Ratio of wine and water = 19 : 81.

13. (b) By the rule of alligation, we have :

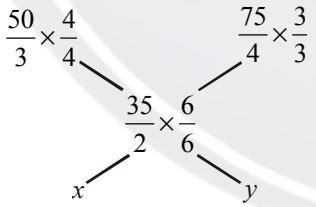


So, ratio of 1st and 2nd quantities = 7 : 14 = 1 : 2.
$\therefore$ Required quantity replaced $= \frac{2}{3}$.

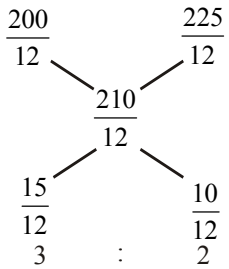
14. (d)



$\Rightarrow$



$\Rightarrow$



$\Rightarrow$ Thus the number of girls = 16 and number of boys = 24

15. (a) Let the number of students be n. Then,

$7 = \frac{n \times 6 + 12 \times 40}{n \quad 12}$

$\Rightarrow$ 7n + 84 = 6n + 480
$\Rightarrow$ n = 480 – 84 = 396

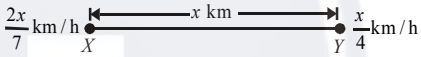
16. (b) Price of the article after first discount
65 – 6.5 = ₹ 58.5
Therefore, the second discount

$= \frac{58.5 - 56.16}{58.5} \times 100 = 4\%$

17. (a) B's income is $\left(\frac{6\frac{1}{4}}{100 + 6\frac{1}{4}} \times 100\right)\%$ less than A's

i.e., $\left(\frac{6.25}{106.25} \times 100\right)\% = 5.89\%$ less than A's income

18. (b) Let the distance between X and Y be x km. Then, the speed of A is $\frac{x}{4}$ km/h and that of B is $\frac{2x}{7}$ km/h.



Relative speeds of the trains

$= \left(\frac{x}{4} + \frac{2x}{7}\right) = \frac{15x}{28}$ km/h

The distance between the trains at 7 a.m. $= x - \frac{x}{2} = \frac{x}{2}$ km

Hence, time taken to cross each other

$= \frac{\frac{x}{2}}{\frac{15x}{28}} = \frac{x}{2} \times \frac{28}{15x} = \frac{14}{15} \times 60 = 56$ min

Thus, both of them meet at 7 : 56 a.m.

19. (c)

Both the athlete are crossing each other after 12 minutes which means the distance between them is 5 km. It will be easy to go through the ratio of the speed which is 2 : 3. The answer is 36 minutes.

20. (c) $x^{\frac{1}{3}} + y^{\frac{1}{3}} = z^{\frac{1}{3}}$ ..... (i)

Taking cube both sides,

$\left(x^{\frac{1}{3}} + y^{\frac{1}{3}}\right)^3 = z$

$\Rightarrow$ $(x + y + 3\ x^{\frac{1}{3}}.y^{\frac{1}{3}}\left(x^{\frac{1}{3}}+y^{\frac{1}{3}}\right) = z$

$[\because (a + b)^3 = a^3 + b^3 + 3ab\ (a + b)]$

$\Rightarrow$ $x + y - z = -3.\ x^{\frac{1}{3}}.y^{\frac{1}{3}}.z^{\frac{1}{3}}$ ..... (ii)

[From equation (i)]

$\therefore$ $(x + y - z)^3 + 27\ xyz$

$= \left(-3x^{\frac{1}{3}}.y^{\frac{1}{3}}.z^{\frac{1}{3}}\right)^3 + 27\ xyz$

[From equation (ii)]

$= -27xyz + 27xyz = 0$

21. (c) Let the annual instalment be ₹ x

By formula $A = P\left(1+\frac{R}{T}\right)^T$

$A = P_1\left(1+\frac{25}{200}\right)$

$\Rightarrow$ $x = P_1 \times \frac{9}{8}$

$\Rightarrow$ $P_1 = \frac{8}{9}x$

Similarly, $= P_2 = \frac{64}{81}x$

$P_1 + P_2 = 6800$

$\Rightarrow$ $\frac{8}{9}x\ \ \frac{64}{81}x = 6800$

$\Rightarrow$ $\frac{72x+64x}{81} = 6800$

$\Rightarrow$ $\frac{136x}{81} = 6800$

$\Rightarrow$ $x = \frac{6800\times 81}{136} = ₹\ 4050$

22. (c) $a+\frac{1}{b}=1 \Rightarrow a = 1-\frac{1}{b} = \frac{b-1}{b}$

$\Rightarrow$ $\frac{1}{a} = \frac{b}{b-1}$ and

$b+\frac{1}{c}=1 \Rightarrow \frac{1}{c} = 1-b \Rightarrow c = \frac{1}{1-b}$

$\therefore$ $c+\frac{1}{a} = \frac{1}{1-b}+\frac{b}{b-1}$

$= \frac{1}{1-b}-\frac{b}{1-b}\ \ \frac{1-b}{1-b}\ \ 1$

23. (d) $\frac{5x}{-15}+\frac{3y}{15}=1 \Rightarrow \frac{x}{-3}+\frac{y}{5}=1$

$\therefore$ area of $\Delta = \frac{1}{2}\times 3\times 5 = \frac{15}{2}$

24. (a) Since, the given lines are parallel, we have

$\frac{1}{k}=\frac{2}{4} \Rightarrow k = 2$

25. (c) $\sec\theta + \tan\theta = p$ or $\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta} = p$

$\Rightarrow$ $1 + \sin\theta = p\cos\theta$

On squaring both sides, we get

$1 + \sin^2\theta + 2\sin\theta = p^2\cos^2\theta$

$1 + 1 - \cos^2\theta + 2\sin\theta$

$= p^2\cos^2\theta$

$2 + 2\sin\theta - \cos^2\theta = p^2\cos^2\theta$

$2(1 + \sin\theta) - \cos^2\theta = p^2\cos^2\theta$

$2 \times p\cos\theta - \cos^2\theta = p^2\cos^2\theta$

$2p\cos\theta = p^2\cos^2\theta + \cos^2\theta$

$\therefore$ $2p\cos\theta = \cos^2\theta\ (1 + p^2)$

$\Rightarrow$ $\cos\theta = \frac{2p}{1+p^2}$

26. (b) $\tan A + \sin A = p$

$\therefore$ $\tan^2 A + \sin^2 A + 2\tan A \sin A = p^2$ ...(i)

Again, $\tan A - \sin A = q$

$\tan^2 A + \sin^2 A - 2\tan A \sin A = q^2$ ...(ii)

On subtracting Eq. (ii) from Eq. (i), we get

$p^2 - q^2 = 4\tan A \sin A$

$\therefore$ $\tan A \sin A = \frac{p^2-q^2}{4}$

Also, $(\tan A - \sin A)(\tan A + \sin A) = pq$

$\Rightarrow$ $\tan^2 A - \sin^2 A = pq$

$\Rightarrow$ $\frac{\sin^2 A}{\cos^2 A} - \sin^2 A = pq$

$\Rightarrow$ $\frac{\sin^2 A(1-\cos^2 A)}{\cos^2 A}\ \ pq$

$\Rightarrow$ $\frac{\sin^2 A \sin^2 A}{\cos^2 A}\ \ pq$

$\Rightarrow$ $\tan^2 A \sin^2 A = pq$

$\therefore$ $\tan A \sin A = \sqrt{pq}$

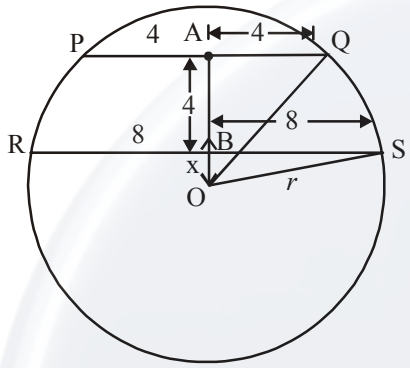
Also, $\tan A \sin A = \frac{p^2-q^2}{4}$

$\therefore \sqrt{pq}\ \ \frac{p^2-q^2}{4}$

$\therefore$ $p^2 - q^2 = 4\sqrt{pq}$

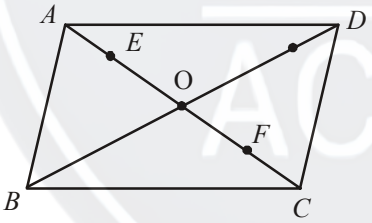
27. (a) Area of original circle
$= \pi \times (6)^2 = 36\pi \text{ cm}^2$
After trisection, the area of the smallest circle
$= \frac{1}{3} \times 36\pi$
$= 12\pi = \pi \times (2\sqrt{3})^2$
$\therefore$ Required radius $= 2\sqrt{3}$ cm

28. (c) In ΔSBO, $r^2 = x^2 + 8^2$ ...(i)
In ΔQAO, $r^2 = (4 + x)^2 + 4^2$ ...(ii)
From Eqs. (i) and (ii), $x^2 + 64 = 16 + x^2 + 8x + 16$



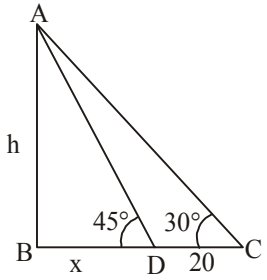
$\Rightarrow \quad 8x = 32 \Rightarrow x = 4$
$\Rightarrow$ From Eq. (i), $r = \sqrt{16 + 64} = \sqrt{80} = 4\sqrt{5}$ cm

29. (a) E and F are the centroids of ΔABD and ΔBCD, so
$\frac{AE}{EO} = \frac{CF}{OF} = \frac{2}{1}$
$AE = CF = 2x$ (say)



Then, $EO = x = OF$
$\Rightarrow \quad EF = 2x$
$\therefore \quad AE = \frac{1}{2} EF = CF$

30. (d)



Let AB be a pillar of height h metre.
If BC = length of shadow $= x + 20$,
DC = 20m
From ΔABD,
$\tan 45° = \frac{h}{x} \Rightarrow h = x$ ..... (i)
From ΔABC,
$\tan 30° = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+20}$
$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{h+20} \Rightarrow \sqrt{3}h = h + 20$
$\Rightarrow \quad (\sqrt{3} - 1)h = 20 \Rightarrow h = \frac{20}{\sqrt{3} - 1}$
$= \frac{20}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$
$= \frac{20(\sqrt{3} + 1)}{2} = 10(\sqrt{3} + 1)$ metre

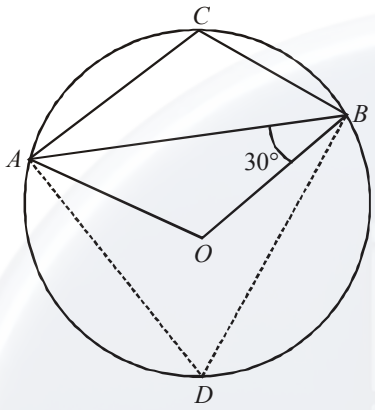
31. (c) $\frac{1}{\text{cosec}^2 51°} + \sin^2 39° + \tan^2 51° - \frac{1}{\sin^2 51°.\sec^2 39°}$
$= \sin^2 51°. + \sin^2 39° + \tan^2 (90° - 39°) - \frac{1}{\sin^2 (90° - 39°).\sec^2 39°}$
$= \cos^2 39° + \sin^2 39° + \cot^2 39° - \frac{1}{\cos^2 39°.\sec^2 39°}$
[$\because \sin (90° - \theta) = \cos \theta$, $\tan (90° - \theta) = \cot \theta$]
$= 1 + \cot^2 39° - 1$
$= \text{cosec}^2 39° - 1 = x^2 - 1$

32. (d) $3 \cot \theta = 4$
$\therefore \cot \theta = \frac{4}{3}$
Now, $\frac{5\sin\theta + 3\cos\theta}{5\sin\theta - 3\cos\theta}$
$= \frac{\frac{5\sin\theta + 3\cos\theta}{\sin\theta}}{\frac{5\sin\theta - 3\cos\theta}{\sin\theta}}$
$\frac{5 + 3\cot\theta}{5 - 3\cot\theta} \quad \frac{5 + 3 \times \frac{4}{3}}{5 - 3 \times \frac{4}{3}}$ 9

33. (b) $\tan x + \cot x = 3$
Squaring on both the sides, we get
$(\tan x + \cot x)^2 = 9$
$\Rightarrow \tan^2 x + \cot^2 x + 2 = 9$
$\Rightarrow (\sec^2 x - 1) + (\text{cosec}^2 x - 1) = 7$
$\therefore \sec^2 x + \text{cosec}^2 x = 9$

34. (d) The greatest value of $(\sin\theta + \cos\theta)$ is $\sqrt{2}$ at $\theta = \frac{\pi}{4}$.

35. (b)



OB = OA = radius of the circle
$\angle AOB = 180 - (30 + 30)$
{Sum of angles of triangle = 180°}
$\Rightarrow 120°$

Then $\angle ADB = \frac{120}{2} = 60°$, because the angle subtended by a chord at the centre is twice of what it can subtend at the circumference. Again, $ACBD$ is a cyclic quadrilateral;
So $\angle ACB = 180° - 60° = 120°$ (because opposite angles of cyclic quadrilateral are supplementary).

36 (a) $\because h = 3$ cm, and $r = 4$ cm

$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 16 \times 3 = 16\pi \text{ cm}^3$

37. (c) $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} - 2$

$= (3 + 2\sqrt{2}) + \frac{1}{3 + 2\sqrt{2}} - 2$

$= (3 + 2\sqrt{2}) + \frac{(3 - 2\sqrt{2})}{(3 + 2\sqrt{2})(3 - 2\sqrt{2})} - 2$

$= (3 \quad 2\sqrt{2}) \quad \frac{(3 - 2\sqrt{2})}{9 - 8} - 2$

$= 3 + 2\sqrt{2} + 3 - 2\sqrt{2} - 2 = 4$

$\therefore \sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{4} = 2$

38. (a) $\frac{a^2 - b^2 - c^2 - 2bc}{a^2 + b^2 - 2ab - c^2}$

$= \frac{a^2 - b^2 \quad c^2 \quad 2bc}{a^2 + b^2 - 2ab \quad - c^2} = \frac{a^2 - (b \quad c)^2}{(a - b)^2 - c^2}$

$= \frac{(a \quad b \quad c)(a - b - c)}{(a - b + c)(a - b - c)}$

$= \frac{a \quad b \quad c}{a - b \quad c} = \frac{0.25 - 0.05 + 0.5}{0.25 + 0.05 + 0.5}$

$= \frac{0.7}{0.8} = \frac{7}{8}$

39. (d) $x^2 - 3x + 1 = 0$

$\therefore \quad x = \frac{3 \pm \sqrt{9 - 4}}{2} = \frac{3 \pm \sqrt{5}}{2}$

$\left[\text{If } ax^2 + bx + c = 0, \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$

Let $x = \frac{3 + \sqrt{5}}{2}$

$\therefore \frac{1}{x} = \frac{2}{3 + \sqrt{5}} = \frac{2(3 - \sqrt{5})}{(3 + \sqrt{5})(3 - \sqrt{5})}$

$= \frac{2(3 - \sqrt{5})}{9 - 5} = \frac{3 - \sqrt{5}}{2}$

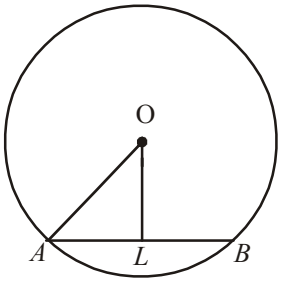
$\therefore x + \frac{1}{x} = \frac{3 + \sqrt{5}}{2} + \frac{3 - \sqrt{5}}{2}$

$= \frac{3 + \sqrt{5} + 3 - \sqrt{5}}{2} = \frac{6}{2} = 3$

When, $x = \frac{3 - \sqrt{5}}{2}$, $\frac{1}{x} = \frac{3 + \sqrt{5}}{2}$

$\therefore x + \frac{1}{x} = \frac{3 - \sqrt{5}}{2} + \frac{3 + \sqrt{5}}{2} = 3$

40. (a)



Let O be the centre of the circle and AB be the chord.
Draw $OL \perp AB$

Then, $AL = \frac{1}{2} \times AB = \left(\frac{1}{2} \times 16\right)$ cm = 8 cm and OA = 10 cm

$OL^2 = OA^2 - AL^2 = (10)^2 - (8)^2$

$= (100 - 64) = 36 = OL = \sqrt{36} = 6$ cm

41. (c) If the number of sides of the regular polygon be n, then

$$\therefore \left(\frac{2n-4}{n}\right) \times 90° = 8 \times \frac{4 \times 90°}{n} + 18$$

$$\Rightarrow \left(\frac{2n-4}{n}\right) \times 5 = \frac{160}{n} + 1$$

$\Rightarrow 10n - 20 = 160 + n$

$\Rightarrow 10n - n = 180$

$\Rightarrow 9n = 180$

$\Rightarrow n = 20$

42. (c) $\therefore \frac{2\pi r_2}{2\pi r_1} = \frac{23}{22}$

$$\Rightarrow \frac{r_2}{r_1} = \frac{23}{22}$$

Let $r_1 = 22 \times x$
and $r_2 = 23 \times x$
$\therefore r_1 - r_2 = 5$
Hence, the diameter will be $110 \times 2 = 220$
$23x - 22x = 5$
$x = 5$
$\therefore$ Inner radius = $22 \times 5 = 110$ m

43. (b) Part of cistern emptied in 1 hour

$$= \frac{1}{5} - \frac{1}{8} \quad \frac{8-5}{40} \quad \frac{3}{40}$$

$\because \frac{3}{40}$ part is emptied in 1 hour.

$\because \frac{3}{4}$ part is emptied n

$\frac{40}{3} \times \frac{3}{5} = 10$ hours.

44. (d) Total number of students playing cricket
$= 38 + 40 + 12 + 17 + 25 + 18 + 20 = 170$

Required percentage $= \frac{25}{170} \times 100 = 14.706 \approx 15\%$

45. (d) Required ratio = 27 : 18 = 3 : 2
46. (b) Looking at the table, we find Cricket is the most popular game.
47. (c) Required ratio = 3 : 2
48. (b) Required difference

$$= \left(\frac{3000+600+2400+1200+3300}{5}\right)$$

$$- \left(\frac{1500+1800+900+2700+1500}{5}\right)$$

$= 2100 - 1680 = 420$ tonnes

49. (a) Required value $= \frac{2700}{1500} = 1.8$

50. (c) Required percentage $= \frac{600}{2400} \times 100 = 25\%$

■■■