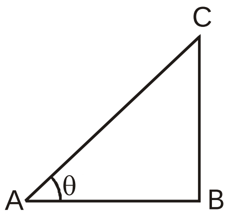
Chapter - 7

HEIGHT AND DISTANCE

Foundation

Solutions

1. (c);



Let BC be the height of the pole and let AB be the length of its shodow.
According to th question:
BC = AB

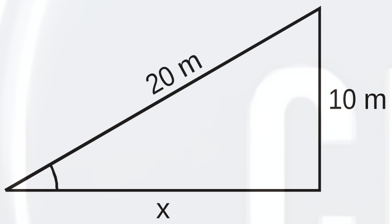
$\Rightarrow \frac{BC}{AB}$

$\Rightarrow \tan\theta = \frac{BC}{AB} = 1$

$\Rightarrow \theta = 45°$

So, the angle of elevation of the sun is 45°.

2. (c);



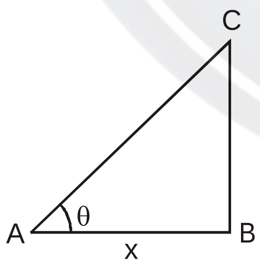
Let the required distance be x.

$\sin\theta = \frac{10}{20} = \frac{1}{2}$

$\Rightarrow \theta = 30°$

$\tan 30° = \frac{10}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{x}, \ x = 10\sqrt{3}$

3. (b);



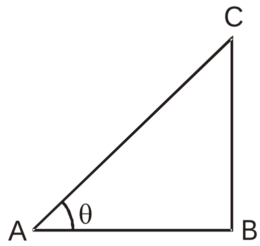
AB = distance between the foot of the ladder and the foot of the wall.
Let AB = x

$\Rightarrow BC = \sqrt{3}x.$

$\tan\theta = \frac{\sqrt{3}x}{x} = \sqrt{3}$

$\Rightarrow \theta = 60°$

4. (b);

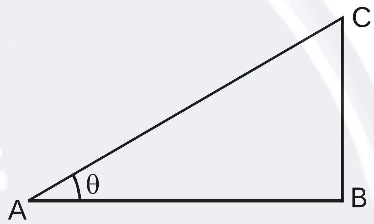


Let BC be the height of the pole and AB be the length of its shodow.
According to the question:
AB = BC

$\Rightarrow \tan\theta = \frac{BC}{AB} = 1$

$\Rightarrow \theta = 45°$

5. (a);



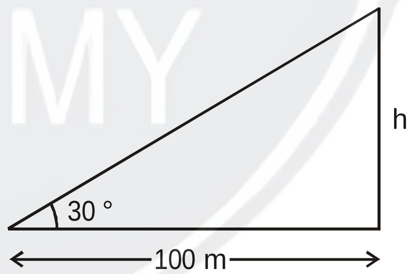
Let BC be the length of the pole and let AB be the length of its shadow.
According to the question:

$AB = \sqrt{3}\,BC$

$\tan\theta = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30°$

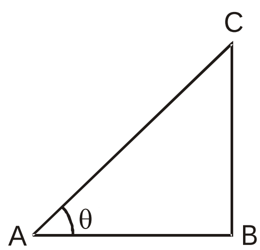
6. (b);



Let the h be the height of the tower.
h = 100 tan 30°

$= \frac{100}{\sqrt{3}}$ m.

7. (d);

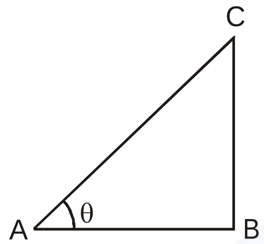


BC is the height of the pillar and AB is the length of the shadow.

$\tan\theta = \frac{BC}{AB} = 1$

$\Rightarrow \quad \theta = 45° = \frac{\pi}{4}$ radians.

8. (c);



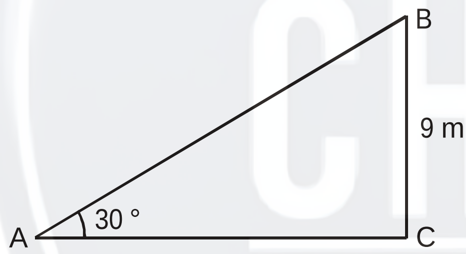
BC = height of the tower
AB = length of its shadow
According to the question:

$AB = \frac{1}{\sqrt{3}} BC$

$\tan\theta = \frac{BC}{AB} = \sqrt{3}$

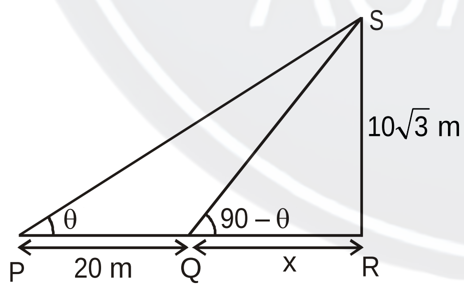
$\Rightarrow \quad \theta = 60°$

9. (c);



$AC = 9 \cot 30° = 9\sqrt{3}$ m.

10. (c);



Let QR = x m.

$\tan\theta = \frac{RS}{PR} = \frac{10\sqrt{3}}{20+x}$

$\tan(90-\theta) = \cot\theta = \frac{10\sqrt{3}}{x}$

$\tan\theta \,.\, \cot\theta = 1$

$\Rightarrow \quad \frac{10\sqrt{3}}{20+x} \cdot \frac{10\sqrt{3}}{x} = 1$

$\Rightarrow \quad x(20+x) = 300$

$x^2 + 20x - 300 = 0$

$\Rightarrow \quad x^2 + 30x - 10x - 300 = 0$

$\Rightarrow \quad (x-10)(x+30) = 0$

$\Rightarrow \quad x = 10$ m.

So, distance of P from the building = 20 + x = 30m.

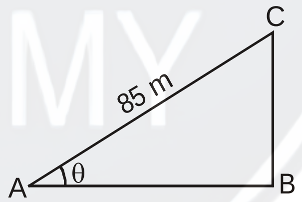
11. (c);

Let the height of the tower be h.

$\frac{h}{15} = \tan 60°$

$\Rightarrow \quad h = 15\sqrt{3}$ m.

12. (b);



$\tan\theta = \frac{8}{15}$

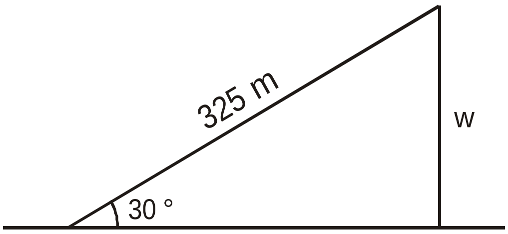
$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+\frac{64}{225}} = \sqrt{\frac{225+64}{225}} = \frac{17}{15}$

$\Rightarrow \quad \cos\theta = \frac{15}{17}$

$\Rightarrow \quad \sin\theta = \sqrt{1-\frac{225}{289}} = \sqrt{\frac{64}{289}} = \frac{8}{17} = \frac{BC}{85}$

$\Rightarrow \quad BC = 40$ m.

13. (d);

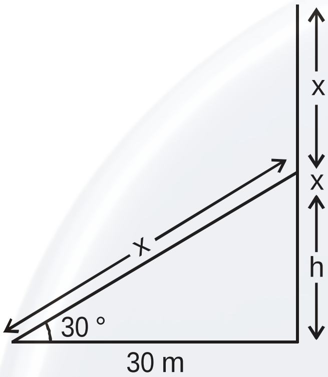


Let the width of the river be w.

$\frac{w}{325} = \sin 30° = \frac{1}{2}$

$\Rightarrow \quad w = \frac{325}{2} = 162.5$ m

14. (b);



$\frac{h}{30} = \tan 30° = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad h = \frac{30}{\sqrt{3}}$ m

$\frac{h}{x} = \sin 30° = \frac{1}{2}$

$\Rightarrow \quad x = 2h$

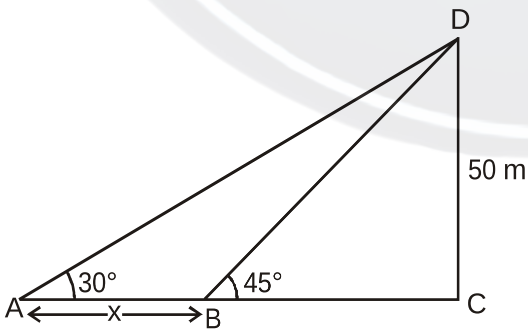
So, height of the tree

$= h + x = h + 2h = 3h$

$= 3 \times \frac{30}{\sqrt{3}}$

$= 30\sqrt{3}$ m.

15. (a);

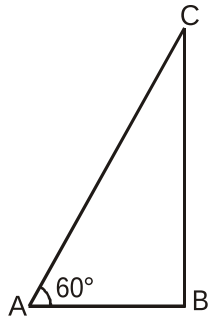


AC = 50 cot 30° = $50\sqrt{3}$ m

BC = 50 cot 45° = 50m

$x = AC - BC = 50\left(\sqrt{3} - 1\right) = 50 \times 0.732 = 36.6$ m.

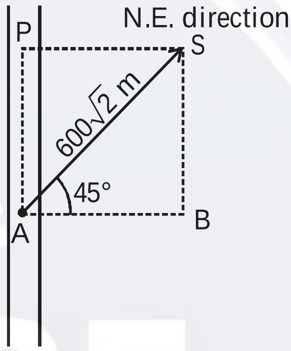
16. (c);



$\frac{BC}{16} = \tan 60° = \sqrt{3}$

$\Rightarrow \quad BC = 16\sqrt{3}$ m.

17. (a);



Perpendicular distance between the school and the highway

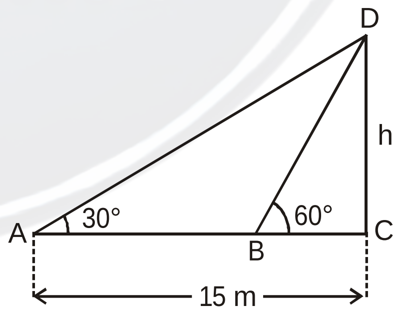
PS = AB

$AB = 600\sqrt{2} \cos 45°$

$= 600\sqrt{2} \times \frac{1}{\sqrt{2}} = 600$ m.

So, PS = 600 m.

18. (c);



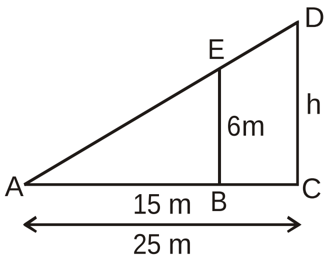
h = 15 tan 30°

$= \frac{15}{\sqrt{3}}$ m.

BC = h cot 60°

$= \frac{15}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = 5$ m.

19. (b);

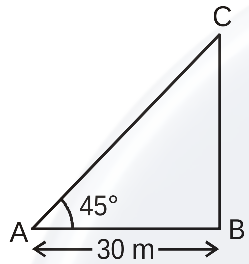


According to the question:

$$\frac{6}{15} = \frac{h}{25}$$

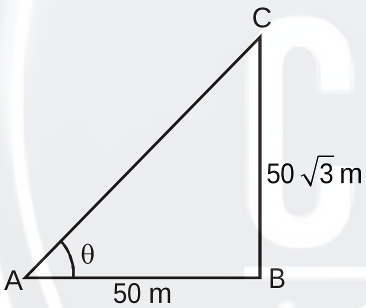
$\Rightarrow$ h = 10 m.

20. (c);



h = 30 tan 45°
= 30 × 1 = 30 m.

21. (a);

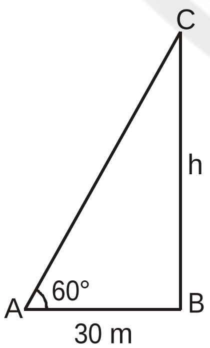


Let the angle of elevation be θ

$$\tan\theta = \frac{50\sqrt{3}}{50} = \sqrt{3}$$

$\Rightarrow$ θ = 60°.

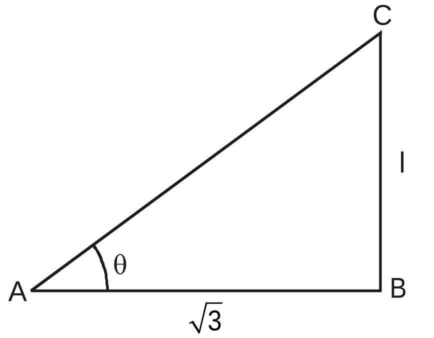
22. (b);



$$\frac{h}{30} = \tan 60° = \sqrt{3}$$

$\Rightarrow$ h = $30\sqrt{3}$ m.

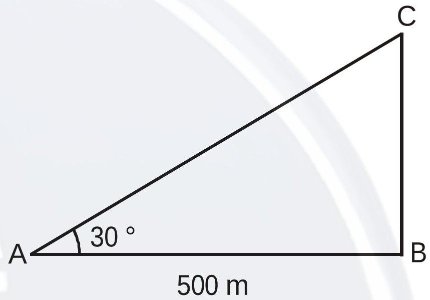
23. (b);



$$\tan\theta = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ θ = 30°

24. (c);



$$\frac{BC}{AB} = \tan 30°$$

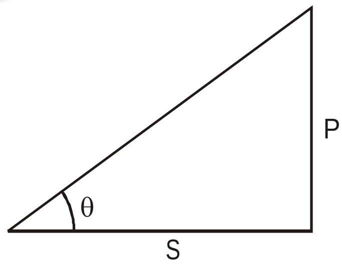
$$BC = 500 \tan 30° = \frac{500}{\sqrt{3}} \text{ m.}$$

25. (d);

Height of the building = BC

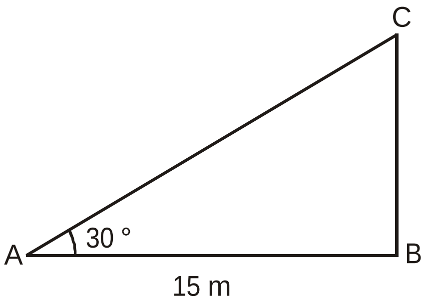
BC = 50 tan 60° = $50\sqrt{3}$ m.

26. (c);



$$P = S\tan\theta = \frac{S}{\cot\theta}$$

27. (c);

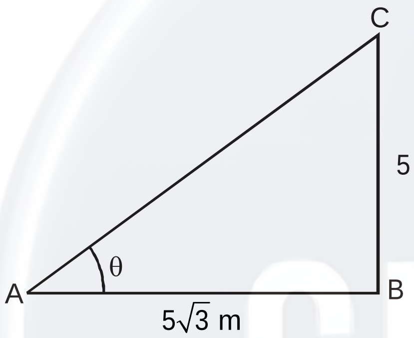


Length of the ladder = AC

$\frac{AB}{AC} = \cos 30° = \frac{\sqrt{3}}{2}$

$AC = \frac{2AB}{\sqrt{3}}\text{ m} = \frac{2 \times 15}{\sqrt{3}}\text{ m.} \quad = 10\sqrt{3}\text{ m}$

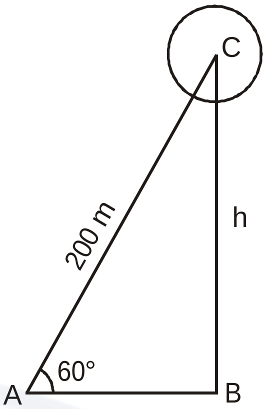
28. (d);



$\tan\theta = \frac{BC}{AB} = \frac{5}{5\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \theta = 30°$

29. (a);



Height of the ballon from the ground = BC.

$BC = 200 \sin 60° = 200 \times \frac{\sqrt{3}}{2} = 100\sqrt{3}\text{ m} = 173.2\text{ m.}$

30. (b);

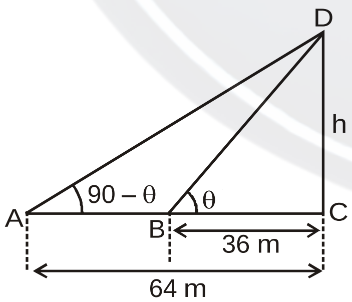
Height of the kite = BC = 50 sin 60°

$= \frac{50\sqrt{3}}{2} = 25\sqrt{3}\text{ m.}$

Moderate

Moderate

1. (b);



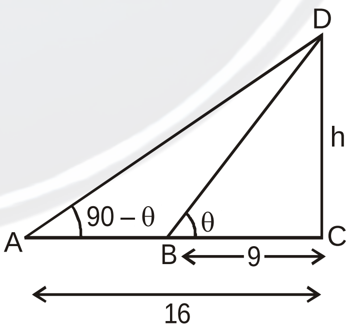
$\tan\theta = \frac{h}{36}$

$\tan(90 - \theta) = \cot\theta = \frac{h}{64}$

$\tan\theta \times \cot\theta = 1$

$\Rightarrow \frac{h}{36} \times \frac{h}{64} = 1 \qquad \Rightarrow \quad h = 6 \times 8 = 48\text{ cm}$

2. (b);

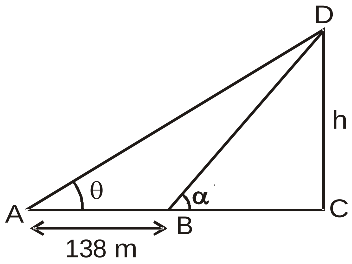


$\frac{h}{9} = \tan\theta, \ \frac{h}{16} = \tan(90 - \theta) = \cot\theta$

$\tan\theta \times \cot\theta = 1$

$\Rightarrow \quad \frac{h}{9} \times \frac{h}{16} = 1 \qquad \Rightarrow \quad h = 3 \times 4 = 12\text{ ft.}$

3. (c);



$\tan\theta = \frac{CD}{AC} = \frac{1}{5}$

$\Rightarrow$ AC = 5CD

$\tan\alpha = \frac{CD}{BC} = \sqrt{\sec^2\alpha - 1}$

$\Rightarrow \frac{CD}{AC-138} = \sqrt{\frac{193}{144} - 1} = \frac{7}{12}$

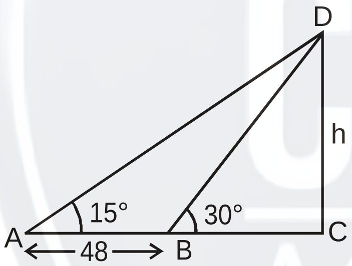
$\Rightarrow AC = \frac{12}{7}CD + 138$ ...(ii)

From (i) and (ii)

$\frac{12}{7}CD + 138 = 5\,CD$

$\Rightarrow$ CD = 42 m

4. (b);



Let the height of the tower be h.

BC = CD Cot 30°

$= \sqrt{3}\,h$

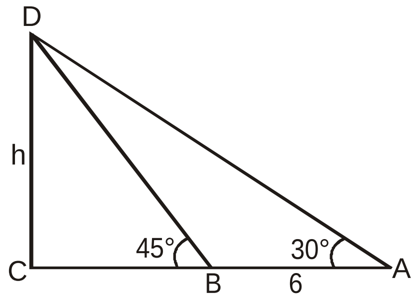
$AC = CD\cot 15° = h(2+\sqrt{3})$

AC – BC = 48 m

$\Rightarrow$ 2h = 48 m

$\Rightarrow$ h = 24 m.

5. (a);



Let the height of the light-house is h.

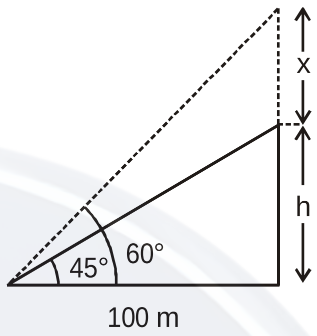
BC = h cot 45° = h

$AC = h\cot 30° = \sqrt{3}\,h$

AB = AC – BC = 6 m

$\Rightarrow (\sqrt{3}-1)h = 6 \quad \Rightarrow h = 3(\sqrt{3}+1)$ m

6. (c);



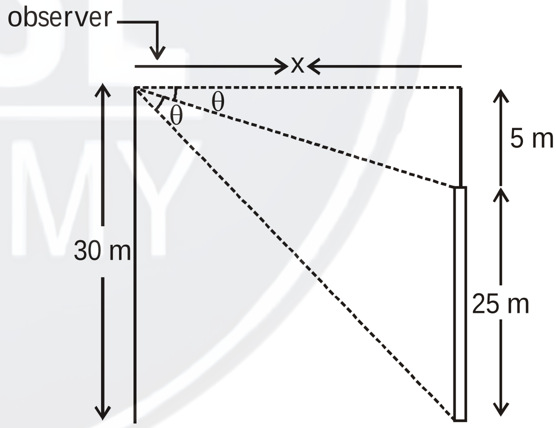
Let the height of the incomplete pillar be h and assume that it has to be increased by x m to complete it.

$\tan 45° = \frac{h}{100} = 1 \Rightarrow h = 100$ m.

$\frac{h+x}{100} = \tan 60° = \sqrt{3}$

$\Rightarrow h + x = 100\sqrt{3}$ m $\Rightarrow x = 100(\sqrt{3}-1)$ m

7. (b);



Let the distance of the observer from the top of the flag staff be x.

According to the question:

$\tan\theta = \frac{5}{x}$

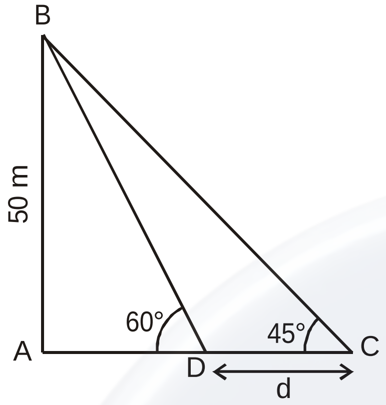
$\tan 2\theta = \frac{30}{x}$

$\Rightarrow \tan 2\theta = 6\tan\theta \Rightarrow \frac{2\tan\theta}{(1-\tan^2\theta)} = 6\tan\theta$

$\Rightarrow \quad 1 - \tan^2\theta = \dfrac{1}{3} \qquad \Rightarrow \quad \tan\theta = \dfrac{\sqrt{2}}{\sqrt{3}} = \dfrac{5}{x}$

$\Rightarrow \quad x = \dfrac{5\sqrt{3}}{\sqrt{2}}.$

8. (b);



Let the distance between the objects be d.

$\dfrac{AB}{AC} = \tan 45° = 1$

$\Rightarrow \quad AC = 50\text{ m}.$

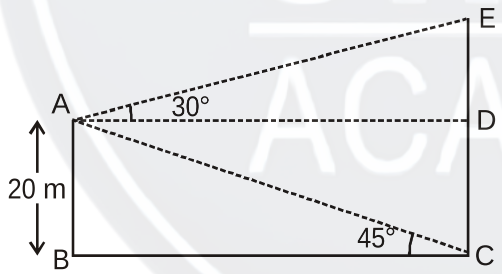
$AD = AB\cot 60° = \dfrac{AB}{\sqrt{3}}.$

The distance between the objects

$= d = AC - AD = 50 - \dfrac{50}{\sqrt{3}}$

$\Rightarrow \quad d = 21\text{m (approx)}.$

9. (a);



$\dfrac{AB}{BC} = \tan 45° = 1$

$\Rightarrow \quad BC = 20\text{ m} \Rightarrow AD = 20\text{ m}$

$\Rightarrow \quad \tan 30° = \dfrac{1}{\sqrt{3}} = \dfrac{DE}{20}$

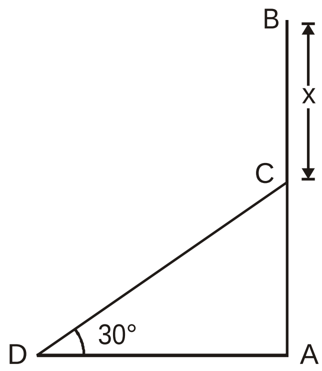
$\Rightarrow \quad DE = \dfrac{20}{\sqrt{3}}\text{ m}$

$CD = AB = 20\text{ m}$

$\Rightarrow$ The height of the second pillar = CD + DE

$= 20 + \dfrac{20}{\sqrt{3}} = \dfrac{20(\sqrt{3}+1)}{\sqrt{3}}$

10. (b);



Let CB be the part of the tree which breaks and takes the position CD.

Let CB = CD = x

AD = 10 m

$\angle ADC = 30°$

$\dfrac{10}{x} = \cos 30° = \dfrac{\sqrt{3}}{2} \qquad \Rightarrow \quad x = \dfrac{20}{\sqrt{3}}\text{ m}$

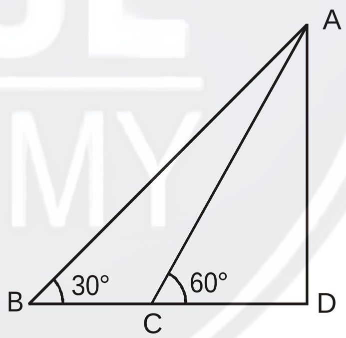
$\dfrac{AC}{AD} = \tan 30° = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad AC = \dfrac{10}{\sqrt{3}}\text{ m}.$

Height of the tree = AC + CB

$= \dfrac{10}{\sqrt{3}} + \dfrac{20}{\sqrt{3}} = 10\sqrt{3}\text{ m}.$

11. (b);



BD = a, CD = b

$\dfrac{AD}{BD} = \tan 30° = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad \dfrac{AD}{a} = \dfrac{1}{\sqrt{3}}$

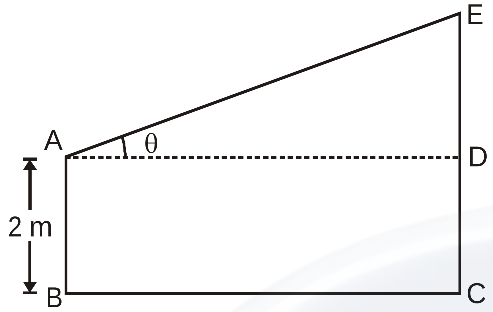
$\dfrac{AD}{CD} = \tan 60° = \sqrt{3}$

$\dfrac{AD}{b} = \sqrt{3}$

$\Rightarrow \quad \frac{AD}{a} \times \frac{AD}{b} = \frac{1}{\sqrt{3}} \times \sqrt{3} = 1$

$\Rightarrow \quad AD = \sqrt{ab}$

12. (b);

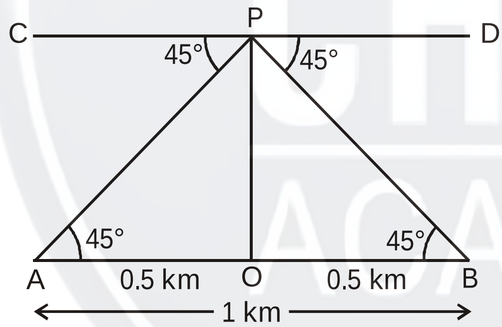


$AD = BC = \frac{4}{\sqrt{3}}\,m$

$DE = \frac{10}{3} - 2 = \frac{4}{3}\,m$

$\tan\theta = \frac{DE}{AD} = \frac{\frac{4}{3}}{\frac{4}{\sqrt{3}}} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \theta = 30°$

13. (b);



Let OP be the width of the river as shown in the diagram.

$\because \quad \angle OAP = \angle APC = 45°$

$\angle OBP = \angle BPD = 45°$

$\therefore \quad \angle OAP = \angle OBP,$

$\therefore \quad AP = BP$

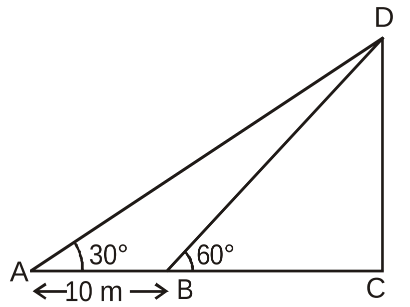
$\Rightarrow$ ΔAPB is an isosceles triangle OP is the distance between the two banks.

$\Rightarrow \quad OP \perp AB$

New, $\frac{OP}{OB} = \tan 45° = 1$

$\Rightarrow OP = 1 \times 0.5 = 0.5$ km.

14. (a);



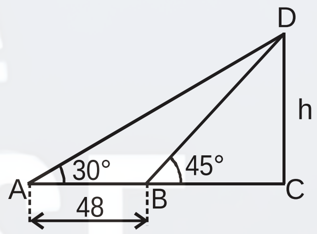
$BC = h \cot 60° = \frac{h}{\sqrt{3}}$

$AC = h \cot 30° = \sqrt{3}\,h$

$AB = AC - BC = \sqrt{3}\,h - \frac{h}{\sqrt{3}} = 10\,m$

$\Rightarrow \quad h = 5\sqrt{3}$ m.

15. (a);



Let the height of the tower be h.

$BC = h \cot 45° = h$

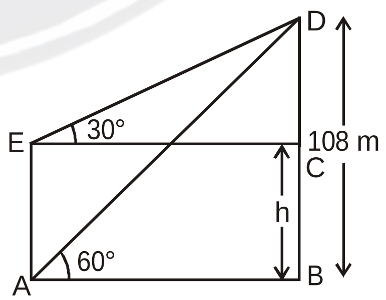
$AC = h \cot 30° = \sqrt{3}\,h$

$AB = \left(\sqrt{3} - 1\right)h = 48\,m$

$h = \frac{48\left(\sqrt{3}+1\right)}{2}$

$h = 24\left(1+\sqrt{3}\right)$ m.

16. (b);



Let the height of the other post be h m.

$\Rightarrow \quad BC = h$ m.

$\Rightarrow \quad CD = (108 - h)$ m.

$$AB = BD \cot 60° = \frac{BD}{\sqrt{3}} = \frac{108}{\sqrt{3}}$$

$$EC = AB = \frac{108}{\sqrt{3}} \text{ m}$$

$$\frac{CD}{EC} = \tan 30° = \frac{1}{\sqrt{3}}$$

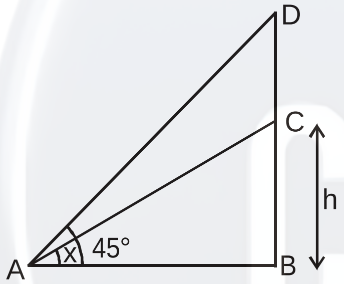
$$\frac{108 - h}{\frac{108}{\sqrt{3}}} = \frac{1}{\sqrt{3}}$$

$$\frac{\sqrt{3}(108 - h)}{108} = \frac{1}{\sqrt{3}}$$

$$108 - h = \frac{108}{3}$$

$$h = 108 - \frac{108}{3} = \frac{2 \times 108}{3} = 72 \text{ m.}$$

17. (b);



$$AB = h \cot x$$

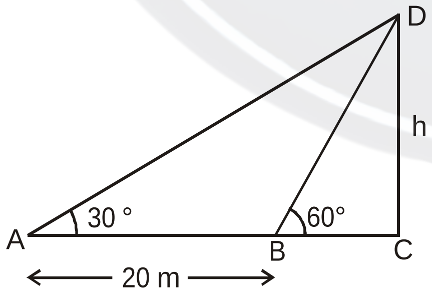
$$\frac{BD}{AB} = \tan 45° = 1$$

$\Rightarrow$ $BD = h \cot x$

The height of the chimney $= CD = BD - h$

$= h \cot x - h.$

18. (c);



Let the height of the tower be h.

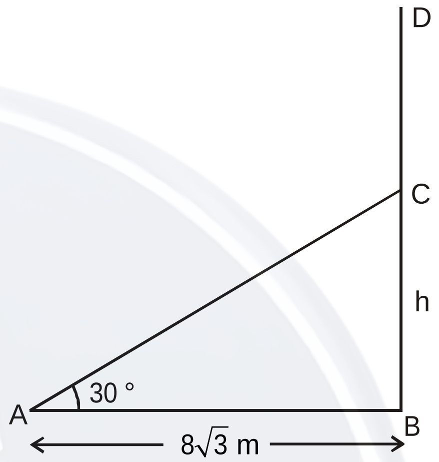
$$AC = h \cot 30° = \sqrt{3}h$$

$$BC = h \cot 60° = \frac{h}{\sqrt{3}}$$

$$AB = AC - BC = \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)h = 20 \text{ m.}$$

$\Rightarrow$ $h = 10\sqrt{3}$ m.

19. (c);



$$BC = AB \tan 30° = 8\sqrt{3} \times \frac{1}{\sqrt{3}}$$

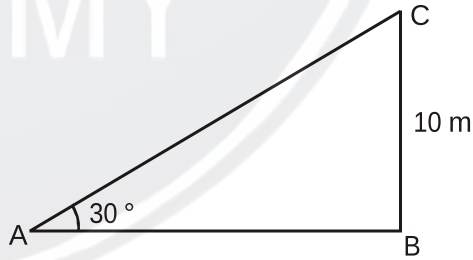
$$\frac{BC}{AC} = \sin 30° = \frac{1}{2}$$

$AC = 2\,BC = 16 = CD$

Height of the post $= BD = BC + CD$

$= 8 + 16 = 24$ m

20. (c);



$$\frac{AB}{BC} = \cot 30° = \sqrt{3}$$

$AB = 10\sqrt{3}$ m

So, the distance of the foot of the ladder from the wall is $10\sqrt{3}$ m