Chapter - 6

TRIGONOMETRY

Foundation

Solutions

1. (a) Clearly, θ lies in 3rd quadrant and in this quadrant $\cos\theta$ is negative

$\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-\frac{144}{169}} = -\sqrt{\frac{25}{169}} = -\frac{5}{13}$

Hence, $\sec\theta = -\frac{13}{5}$

2. (b) $\sin\theta = \frac{5}{13}$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{5}{12}$ and $\sec\theta = \frac{1}{\cos\theta} = \frac{13}{12}$

3. (d) $\sin\theta = \frac{5}{13}$

$\Rightarrow \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$

$\Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{5}{12} = \sqrt{(1+\tan\theta)(1-\tan\theta)}$

$= \sqrt{\frac{119}{144}} = \frac{\sqrt{119}}{12}$

4. (c) $1 + \tan^2\theta = \sec^2\theta$

$\Rightarrow \sec\theta = +\sqrt{1+\tan^2\theta}$ $[0° < \theta < 90°]$

$\Rightarrow \sec\theta = \sqrt{1+\frac{9}{16}} = \sqrt{\frac{25}{16}} = +\frac{5}{4}$

$\Rightarrow \cos\theta = \frac{4}{5}$

$\therefore \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

$\sin\theta.\cos\theta = \frac{3}{5}\times\frac{4}{5} = \frac{12}{25}$

5. (d) $\tan\theta = \frac{3}{4}$

$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+\frac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$

$\Rightarrow \cos\theta = \frac{4}{5}$

$\Rightarrow \sin\theta = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

$\cos^2\theta - \sin^2\theta = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

6. (a); $\frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta = \frac{7}{8}$

7. (b); $(\sin A + \cos A)^2 + (\sin A - \cos A)^2$

$= \sin^2 A + \cos^2 A + 2\sin A\cos A + \sin^2 A + \cos^2 A - 2\sin A\cos A$

$= 2(\sin^2 A + \cos^2 A) = 2$

8. (b) $\sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}} = \sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}\times\frac{\sec x - \tan x}{\sec x - \tan x}}$

$= \sqrt{\frac{(\sec x - \tan x)^2}{\sec^2 x - \tan^2 x}}$

$= \sqrt{(\sec x - \tan x)^2} = \sec x - \tan x$

9. (d) $(1-\tan\theta)^2 + (1+\tan\theta)^2$

$= 1 + \tan^2\theta - 2\tan\theta + 1 + \tan^2\theta + 2\tan\theta$

$= 2(1+\tan^2\theta) = 2\sec^2\theta$

10. (d) $\cos^4\theta - \sin^4\theta = (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

$= (\cos^2\theta - \sin^2\theta)$

11. (c); $\frac{\sin\theta(1-\cos\theta)}{\sin^2\theta} = \frac{1-\cos\theta}{\sin\theta}$

12. (a); $= \frac{(5\cos\theta-4)(4+5\cos\theta)-(3+5\sin\theta)(3-5\sin\theta)}{(3-5\sin\theta)(4+5\cos\theta)}$

$= \frac{[[20\cos\theta+25\cos^2\theta-16-20\cos\theta] - [9-15\sin\theta+15\sin\theta-25\sin^2\theta]]}{(3-5\sin\theta)(4+5\cos\theta)}$

$= \frac{[20\cos\theta+25\cos^2\theta-16-20\cos\theta - 9+15\sin\theta-15\sin\theta+25\sin^2\theta]}{(3-5\sin\theta)(4+5\cos\theta)} = 0$

13. (c); $\cot\theta.\operatorname{cosec}\theta.\sin\theta.\tan\theta$

$= \frac{\cos\theta}{\sin\theta}.\frac{1}{\sin\theta}.\sin\theta.\frac{\sin\theta}{\cos\theta} = 1$

14. (c); $\left(\frac{\tan A}{\sec A-1}+\frac{\tan A}{\sec A+1}\right)$

$= \frac{\tan A(\sec A+1)+\tan A(\sec A-1)}{(\sec A-1)(\sec A+1)}$

$= \frac{\tan A\sec A+\tan A+\tan A\sec A-\tan A}{\sec^2 A-1}$

$= \frac{2\tan A\sec A}{\tan^2 A} = \frac{2\sec A}{\tan A} = 2\frac{1}{\cos A}\times\frac{\cos A}{\sin A}$

$= 2\,\text{cosec}\,A$

15. (d) $\frac{\sin A+\sin B}{\cos A+\cos B}+\frac{\cos A-\cos B}{\sin A-\sin B}$

$= \frac{\sin^2 A-\sin^2 B+\cos^2 A-\cos^2 B}{(\cos A+\cos B)(\sin A-\sin B)}$

$= \frac{(\sin^2 A+\cos^2 A)-(\sin^2 B+\cos^2 B)}{(\cos A+\cos B)(\sin A-\sin B)} = 0$

16. (c) $a\cos\theta + b\sin\theta = 4$, $a\sin\theta - b\cos\theta = 3$.

Squaring and adding, we get

$a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta$

$= 4^2 + 3^2 = 25$

$a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$

$= 25$

$\Rightarrow \quad a^2 + b^2 = 25$

17. (b); $\sec^2 60° - 1 = 2^2 - 1 = 3$

18. (b); $\sin^2 30° + 4\cot^2 45° - \sec^2 60°$

$= \left(\frac{1}{2}\right)^2 + 4\times 1 - 2^2 = \frac{1}{4}+4-4 = \frac{1}{4}$

19. (a); $x\tan 45°\cos 60° = \sin 60°\cot 60°$

$x\times 1\times\frac{1}{2} = \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{3}}$, $x = 1$

20. (d); $\sin\frac{\pi}{6}+\cos\frac{\pi}{3}+\tan\frac{\pi}{4} = \frac{1}{2}+\frac{1}{2}+1 = 2$

21. (d); $\sin^2 20° + \sin^2 40° + \sin^2 50° + \sin^2 70°$

$= \sin^2 20° + \sin^2 70° + \sin^2 40° + \sin^2 50°$

$= \sin^2 20° + \sin^2(90° - 20°) + \sin^2 40° + \sin^2(90° - 40°)$

$= (\sin^2 20° + \cos^2 20°) + (\sin^2 40° + \cos^2 40°)$

$= 1 + 1 = 2 = p$.

22. (c) $\sin^2 15° + \sin^2 30° + \sin^2 45° + \sin^2 60° + \sin^2 75°$

$= \sin^2 15° + \sin^2 75° + \sin^2 30° + \sin^2 60° + \sin^2 45°$

$= (\sin^2 15° + \cos^2 15°) + (\sin^2 30° + \cos^2 30°) + \sin^2 45°$

$= 1+1+\frac{1}{2} = \frac{5}{2} = 2.5$

23. (a) $\frac{x\,\text{cosec}^2 30°\sec^2 45°}{8\cos^2 45°\sin^2 60°} = \tan^2 60° - \tan^2 30°$

$= \frac{x\times 2^2\times(\sqrt{2})^2}{8\times\left(\frac{1}{\sqrt{2}}\right)^2\times\left(\frac{\sqrt{3}}{2}\right)^2} = (\sqrt{3})^2 - \left(\frac{1}{\sqrt{3}}\right)^2$

$= \frac{8x}{8\times\frac{1}{2}\times\frac{3}{4}} = 3-\frac{1}{3}, \quad \frac{8x}{3} = \frac{8}{3}$

$\Rightarrow x = 1$

24. (a); $\cos^2 15° + \cos^2 75°$

$\cos^2(90° - 75°) + \cos^2(90° - 15°)$

$\sin^2 75° + \sin^2 15°$

25. (a); $A + B = 180°$

$\Rightarrow \quad A = 180° - B$

$\Rightarrow \quad \sin A = \sin(180° - B) = \sin B$

26. (c); $\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$

$\tan(A+B) = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}} = \frac{\frac{5}{6}}{1-\frac{1}{6}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$

$\tan(A+B) = 1$

$\Rightarrow \quad A + B = \frac{\pi}{4}$.

27. (b); $\tan(x+y) = \frac{\tan x+\tan y}{1-\tan x\tan y}$

$= \frac{\frac{m}{m+1}+\frac{1}{2m+1}}{1-\frac{m}{m+1}\cdot\frac{1}{2m+1}} = \frac{m(2m+1)+(m+1)}{(m+1)(2m+1)-m}$

$= \frac{2m^2+2m+1}{2m^2+2m+1} = 1, \ \tan(x+y) = 1$

$\Rightarrow \quad x + y = \frac{\pi}{4}$

28. (d) $\sin(45° + \theta) - \sin(45° - \theta)$
$\sin 45° \cos\theta + \cos 45° \sin\theta - [\sin 45° \cos\theta - \cos 45° \sin\theta]$
$= \sin 45° \cos\theta + \cos 45° \sin\theta - \sin 45° \cos\theta + \cos 45° \sin\theta$
$= 2\cos 45° \sin\theta = 2\times\frac{1}{\sqrt{2}}\cdot\sin\theta = \sqrt{2}\sin\theta$.

29. (b) $\sin 2x = \frac{1}{5}$ (given)
$(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x.\cos x$
$= 1 + \sin 2x$
$\Rightarrow (\sin x + \cos x)^2 = 1 + \frac{1}{5} = \frac{6}{5}$
$\Rightarrow \sin x + \cos x = \sqrt{\frac{6}{5}}$

30. (d) Maximum value of $\cos 2x = 1$
Minimum value of $\cos 2x = -1$
$\Rightarrow 0 \le (1 + \cos^2 x) \le 2$

31. (d) $\sin x = \frac{1}{2}$
$\Rightarrow \cos x = \sqrt{1-\sin^2 x} = \sqrt{1-\left(\frac{1}{2}\right)^2} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$
$\Rightarrow \tan x = \frac{1}{\sqrt{3}}, \tan 2x = \frac{2\tan x}{1-\tan^2 x}$
$= \frac{2\times\frac{1}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3}$

32. (a) $\sin\theta + \cos\theta = \sqrt{2}$
$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$
$1 + \sin 2\theta = 2, \quad \sin 2\theta = 1$
$\Rightarrow 2\theta = 90°$
$\Rightarrow \theta = 45° = \frac{\pi}{4}$.

33. (d) $2\sin^2\theta = 3\sin\theta - 1$
$2\sin^2\theta - 3\sin\theta + 1 = 0$
$2\sin^2\theta - 2\sin\theta - \sin\theta + 1 = 0$
$2\sin\theta(\sin\theta - 1) - 1(\sin\theta - 1) = 0$
$\Rightarrow (\sin\theta - 1)(2\sin\theta - 1) = 0$
$\Rightarrow \sin\theta = 1 \text{ or } \sin\theta = \frac{1}{2}, \quad \theta = \frac{\pi}{2}, \frac{\pi}{6}$

34. (c) $(1-\sin^2\alpha)(\tan^2\alpha) = \cos^2\alpha\cdot\frac{\sin^2\alpha}{\cos^2\alpha} = \sin^2\alpha$.

35. (a) When $p = q$,
$\sin^2\theta = \frac{2q^2}{2q^2} = 1$, which is possible

When, $p = -q$, $\sin^2\theta = \frac{2q^2}{-2q^2}$
$= -1$, which is not possible

When, $2p = 3q$, $p = \frac{3q}{2}$
$\sin^2\theta = \frac{\frac{9}{4}q^2 + q^2}{2\times\frac{3q}{2}\cdot q} = \frac{\frac{13q^2}{4}}{3q^2} = \frac{13}{12}$,

which is not not possible, when, $p = 2q$,
$\sin^2\theta = \frac{4q^2 + q^2}{2\cdot 2q\cdot q} = \frac{5q^2}{4q^2}$
$\Rightarrow \sin^2\theta = \frac{5}{4}$, which is not possible
So, correct option is (a).

36. (c) $\sin\theta = \frac{4}{5}$
$\cos\theta = \sqrt{1-\sin^2\theta} = -\sqrt{1-\sin^2\theta}$
(because θ is in second quadrant)
$= -\sqrt{1-\frac{16}{25}} = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$

37. (b) $\sin\alpha + \cos\beta = 2$
maximum value of $\sin\theta$ is 1 and maximum value of $\cos\theta$ is 1.
$\Rightarrow \sin\alpha = \cos\beta = 1$
$\Rightarrow \alpha = 90°, \beta = 0°$
$\sin\left(\frac{2\alpha+\beta}{3}\right) = \sin\left(\frac{180°}{3}\right)$
$\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 30° = \cos\left(\frac{\alpha}{3}\right)$

38. (c) $\cos^4\theta - \sin^4\theta = \frac{2}{3}$

$(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta) = \frac{2}{3}$

$\Rightarrow \cos^2\theta - \sin^2\theta = \frac{2}{3}$

$\Rightarrow \cos^2\theta - (1 - \cos^2\theta) = \frac{2}{3}$

$\Rightarrow 2\cos^2\theta - 1 = \frac{2}{3}$

39. (a) $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = \frac{1-\sin\theta+1+\sin\theta}{1-\sin^2\theta}$

$= \frac{2}{\cos^2\theta} = 2\sec^2\theta$

40. (a) $\cos^2\theta(1+\tan^2\theta) = \cos^2\theta\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)$

$= \cos^2\theta + \sin^2\theta = 1$

41. (d) $(\sec^2\theta - \tan^2\theta) = 1$

trignometric identity and hence it is valid for all values of θ.

42. (b) $\tan x = \frac{4}{3}\ \sqrt{\frac{(1-\sin x)(1+\sin x)}{(1+\cos x)(1-\cos x)}}$

$= \sqrt{\frac{1-\sin^2 x}{1-\cos^2 x}} = \sqrt{\frac{\cos^2 x}{\sin^2 x}} = \frac{\cos x}{\sin x} = \cot\ x = \frac{3}{4}$

43. (a) $\sec^4\theta - \tan^4\theta = (\sec^2\theta + \tan^2\theta)(\sec^2\theta - \tan^2\theta)$

$= \sec^2\theta + \tan^2\theta = \frac{7}{12}$

44. (b) $\sin^2 60° + \cos^2(3x - 9°) = 1$

$\sin^2\theta + \cos^2\theta = 1, \quad 3x - 9° = 60°$

$3x = 69°, \quad x = 23°$

45. (b) $\sin(x-y) = \frac{1}{2} = \sin 30°$

$x - y = 30°$ ... (i)

$\cos(x+y) = \frac{1}{2} = \cos 60°$

$x + y = 60°$ ... (ii)

Adding (i) and (ii) :

$2x = 90°$

$x = 45°$ ... (iii)

from (ii) and (iii) : $y = 15°$

46. (b) $(\sec^2\theta - 1)(\mathrm{cosec}^2\theta - 1) = \tan^2\theta \times \cot^2\theta = 1$

47. (c) $2\cos 3\theta_1 = 1, \quad \cos 3\theta_1 = \frac{1}{2} = \cos 60°$

$3\theta_1 = 60°, \quad \theta_1 = 20°, \quad 2\sin 2\theta_2 = \sqrt{3}$

$\sin 2\theta_2 = \frac{\sqrt{3}}{2} = \sin 60°, \quad 2\theta_2 = 60°, \quad \theta_2 = 30°$

48. (d) $\tan(\alpha+\beta) = 1; \tan(\alpha-\beta) = \frac{1}{\sqrt{3}}$

$\alpha + \beta = 45; \alpha - \beta = 30°, \quad 2\alpha + 2\beta = 90°$

49. (d) $-\sqrt{a^2+b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$

$\Rightarrow$ minimum value of $(\sin\theta + \cos\theta)$

is $-\sqrt{1^2+1^2} = -\sqrt{2}$

50. (c) $\sin\alpha = \sin\beta, \quad \cos\alpha = \cos\beta, \quad \sin\alpha - \sin\beta = 0$

$2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) = 0$

$\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) = 0$ ... (i)

$\cos\alpha - \cos\beta = 0$

$2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right) = 0$

$-\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) = 0$ ... (ii)

form (i) and (ii)

$\sin\left(\frac{\alpha-\beta}{2}\right)\left[\cos\left(\frac{\alpha+\beta}{2}\right) - \sin\left(\frac{\alpha+\beta}{2}\right)\right] =$

$\sin\left(\frac{\alpha-\beta}{2}\right) = 0$ or $\cos\left(\frac{\alpha+\beta}{2}\right) = \sin\left(\frac{\alpha+\beta}{2}\right)$

$\tan\left(\frac{\alpha+\beta}{2}\right) = 1, \quad \frac{\alpha+\beta}{2} = 45°, \quad \alpha + \beta = 90°$

$\sin\alpha = \sin\beta$ and $\alpha + \beta = 90°, \quad \alpha = \beta = 45°$

so, $\sin\left(\frac{\alpha-\beta}{\sqrt{2}}\right) = \sin(0°) = 0$

Moderate

1. (a) $\tan\theta = \frac{x}{y}$, $\frac{x\sin\theta + y\cos\theta}{x\sin\theta - y\cos\theta}$

$= \frac{x\frac{\sin\theta}{\cos\theta} + y}{x\frac{\sin\theta}{\cos\theta} - y} = \frac{x\tan\theta + y}{x\tan\theta - y}$

$= \frac{x\times\frac{x}{y} + y}{x\times\frac{x}{y} - y} = \frac{x^2+y^2}{x^2-y^2}$

2. (c) $7\sin^2\theta + 3\cos^2\theta = 4$

$3\sin^2\theta + 3\cos^2\theta = 4 - 4\sin^2\theta$

$3 \times 1 = 4(1-\sin^2\theta) = \cos^2\theta = \frac{3}{4}$

$\cos\theta = \frac{\sqrt{3}}{2}$, $\theta = 30$, $\tan\theta = \frac{1}{\sqrt{3}}$

3. (d) $\sec^2A = 3$

$\tan^2A = \sec^2A - 1 = 2$

$\cot^2A = \frac{1}{\tan^2A} = \frac{1}{2}$

$\text{cosec}^2A = 1 + \cot^2A = 1 + \frac{1}{2} = \frac{3}{2}$

$\frac{\tan^2A - (\text{cosec}^2A)}{\tan^2A + \text{cosec}^2A} = \frac{2-\frac{3}{2}}{2+\frac{3}{2}} = \frac{1}{7}$

4. (a) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sqrt{\frac{1+\sin A}{1-\sin A}\times\frac{1+\sin A}{1+\sin A}}$

$= \sqrt{\frac{(1+\sin A)^2}{(1-\sin^2A)}} = \sqrt{\frac{(1+\sin A)^2}{\cos^2A}} = \frac{1+\sin A}{\cos A}$

$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$

5. (a) $\sec^4A - \sec^2A$

$= \sec^2A(\sec^2A - 1) = \sec^2A \cdot \tan^2A$

$= (1+\tan^2A)(\tan^2A) = \tan^2A + \tan^4A$

6. (b) $\cot^4\theta - \text{cosec}^4\theta + \cot^2\theta + \text{cosec}^2\theta$

$= \cot^4\theta + \cot^2\theta - \text{cosec}^4\theta + \text{cosec}^2\theta$

$= \cot^2\theta(\cot^2\theta + 1) - \text{cosec}^2\theta(\text{cosec}^2\theta - 1)$

$= \cot^2\theta \cdot \text{cosec}^2\theta - \text{cosec}^2\theta \cdot \cot^2\theta = 0$

7. (a) $\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta} = \frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\sin^2\theta}{\sin\theta - \cos\theta} + \frac{\cos^2\theta}{\cos\theta - \sin\theta}$

$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta} = \sin\theta + \cos\theta$

8. (b) $\frac{\cot\theta}{\cot\theta - \cot 3\theta} + \frac{\tan\theta}{\tan\theta - \tan 3\theta}$

$\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta} - \frac{\cos 3\theta}{\sin 3\theta}} + \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} - \frac{\sin 3\theta}{\cos 3\theta}}$

$= \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta.\sin 3\theta - \sin\theta\cos 3\theta}{\sin\theta\sin 3\theta}} + \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta\cos 3\theta - \sin 3\theta\cos\theta}{\cos\theta\cos 3\theta}}$

$= \frac{\cos\theta}{\sin\theta}.\frac{\sin\theta\sin 3\theta}{\cos\theta.\sin 3\theta - \sin\theta\cos 3\theta} + \frac{\sin\theta}{\cos\theta}.\frac{\cos\theta\cos 3\theta}{\sin\theta\cos 3\theta - \sin 3\theta\cos\theta}$

$= \frac{\cos\theta\sin 3\theta - \sin\theta\cos 3\theta}{\cos\theta\sin 3\theta - \sin\theta\cos 3\theta} = 1$

9. (c); $\sin\theta + \cos\theta = m$

squaring on both side

$\sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta = m^2$

$1 + 2\sin\theta.\cos\theta = m^2$

$\sin\theta.\cos\theta = \frac{m^2-1}{2}$

$\sin^3\theta + \cos^3\theta = n$

$(\sin\theta + \cos\theta)^3 - 3\sin\theta.\cos\theta(\sin\theta + \cos\theta) = n$

Using equation (i) and (ii)

$(m)^3 - 3\left(\frac{m^2-1}{2}\right)(m) = n$

$2m^3 - 3(m^3 - m) = 2n$, $2m^3 - 3m^3 + 3m = 2n$

$m^3 - 3m + 2n = 0$

10. (c); $\cot 75° = \cot(90° - 15°) = \tan 15°$
similarly,
$\cot 74° = \tan 16°$, $\cot 73° = \tan 17°$
$\cot 45° = 1$
$\therefore$ $(\cot 15° \cot 75°)(\cot 16° \cot 74°)$ .... $(\cot 45°)$
$= (\cot 15°.\tan 15°).(\cot 16°.\tan 16°)$ .... $(\cot 45°.\tan 45°)$
$= 1 \times 1 \times$ .... $\cot 45° = 1$

11. (a); $\cos 11° = \cos(90° - 79°) = \sin 79°$
$\sin 11° = \sin(90° - 79°) = \cos 79°$.
$\sin 79° \cos 11° + \sin 11° \cos 79°$
$= \sin 79° . \sin 79° + \cos 79° . \cos 79°$
$= \sin^2 79° + \cos^2 79° = 1$

12. (c) $\cos 160° = \cos(180° - 20°) = -\cos 20°$
$\cos 140° = \cos(180° - 40° = -\cos 40°$
similarly, $\cos 120° = -\cos 60°$
$\cos 20° + \cos 40° + \cos 60° + ... + \cos 160° + \cos 180°$
$= (\cos 20° + \cos 160°) + (\cos 40° + \cos 140°) + (\cos 60° + \cos 120°) + (\cos 80° + \cos 100°) + \cos 180°$
$= (\cos 20° - \cos 20°) + (\cos 40° - \cos 40°) + (\cos 60° - \cos 60°) + (\cos 80° - \cos 80°) + (\cos 180°)$
$= -1$

13. (a) $\tan \frac{x}{2} = \sqrt{\frac{1-\cos x}{1+\cos x}}$
for $x = 45°$
$\tan 22° 30' = \sqrt{\frac{1-\cos 45°}{1+\cos 45°}}$
$$= \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}} = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}}$$
$$= \sqrt{\frac{(\sqrt{2}-1)^2}{1}} = \sqrt{2}-1$$

14. (a) $4 \sin 45° \cos 15°$
$= 2[\sin(45° + 15°) + \sin(45° - 15°)]$
$$= 2[\sin 60° + \sin 30°] = 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = \sqrt{3}+1$$

15. (b) $\sin\theta = \frac{1}{2} \Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$
$\cos\phi = 1 \Rightarrow \sin\phi = 0$
$$\cot(\theta + \phi) = \frac{\cos(\theta+\phi)}{\sin(\theta+\phi)} = \frac{\cos\theta\cos\phi - \sin\theta\sin\phi}{\sin\theta\cos\phi + \cos\theta\sin\phi}$$
$$= \frac{\frac{\sqrt{3}}{2}\times 1 - \frac{1}{2}\times 0}{\frac{1}{2}\times 1 + \frac{\sqrt{3}}{2}\times 0} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

16. (d) ABCD is cyclic quadrilateral
$A + C = B + D = \pi$
$A = \pi - C \Rightarrow \cos A = \cos(\pi - C) = -\cos C$
$\cos A + \cos C = 0$
similarly, $\cos B + \cos D = 0$
$\cos A + \cos C - (\cos B + \cos D) = 0$
$= \cos A - \cos B + \cos C - \cos D = 0$

17. (c) $\frac{1+\tan 15°}{1-\tan 15°} = \frac{\tan 45° + \tan 15°}{1 - \tan 45°.\tan 15°}$
$= \tan(45° + 15°) = \tan 60° = \sqrt{3}$
$$\left[\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

18. (d) $\cos A - \sin A > 0$, $\sin A < \cos A$, $\tan A < 1$, $A < 45°$
$\therefore \sin A + \cos A < \sin 45° + \cos 45°$, $= \sqrt{2}$

19. (a) $\frac{\cot A + (\operatorname{cosec} A - 1)}{\cot A - (\operatorname{cosec} A - 1)} \times \frac{\cot A + (\operatorname{cosec} A - 1)}{\cot A + (\operatorname{cosec} A - 1)}$
$$= \frac{\cot^2 A + (\operatorname{cosec} A - 1)^2 + 2\cot A(\operatorname{cosec} A - 1)}{\cot^2 A - (\operatorname{cosec} A - 1)^2}$$
$$= \frac{(\operatorname{cosec}^2 A - 1) + (\operatorname{cosec} A - 1)^2 + 2\cot A(\operatorname{cosec} A - 1)}{\cot^2 A - \left[\operatorname{cosec}^2 A + 1 - 2\operatorname{cosec} A\right]}$$
$$= \frac{2(\operatorname{cosec} A - 1)(\operatorname{cosec} A + \cot A)}{2(\operatorname{cosec} A - 1)}$$
$= \operatorname{cosec} A + \cot A$

20. (a); $\cos 3\theta + \sin 3\theta = \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos 3\theta + \frac{1}{\sqrt{2}}\sin 3\theta\right)$
$= \sqrt{2}(\sin 45° \cos 3\theta + \cos 45° \sin 3\theta)$
$= \sqrt{2}\sin(45° + 3\theta)$
maximum value occurs when,
$\sin(45° + 3\theta) = 1$
$\Rightarrow$ $3\theta = 45°$ or $\theta = 15°$

21. (a); Given $= \frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°}$
$$\cong \frac{\frac{\cos 11° + \sin 11°}{\cos 11°}}{\frac{\cos 11° - \sin 11°}{\cos 11°}} = \frac{1 + \tan 11°}{1 - \tan 11°}$$
$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$ $(\because \tan 45° = 1)$
$\cong \tan(45 + 11) = \tan 56°$

22. (d) $\sin^2\left(\frac{\pi}{8}+\frac{A}{2}\right)-\sin^2\left(\frac{\pi}{8}-\frac{A}{2}\right)$

$=\frac{1}{2}\left[2\sin^2\left(\frac{\pi}{8}+\frac{A}{2}\right)-2\sin^2\left(\frac{\pi}{8}-\frac{A}{2}\right)\right]$

$=\frac{1}{2}\left[1-\cos\left(\frac{\pi}{4}+A\right)-1+\cos\left(\frac{\pi}{4}-A\right)\right]$

[$\because$ $2\sin^2\theta = 1-\cos 2\theta$]

$=\frac{1}{2}\left[2\sin\frac{\pi}{4}\sin A\right]=\frac{1}{\sqrt{2}}\sin A$

23. (a) Clearly, $\sin\theta$ is increasing in 1st quadrant and

$0<\sin\theta<\cos\theta$ when $0<\theta<\frac{\pi}{4}$.

So, statements 1. and 3. are correct.
clearly, statement 4 is incorrect,
Since,

$\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}>\frac{1}{2}=\cos\frac{\pi}{3}$

24. (c) $\tan x=\frac{1}{\sqrt{3}}$, $x=30°$

$\sin y=\frac{1}{\sqrt{2}}$, $y=45°$

$x+y=30°+45°=75°$
So, x + y lies between 0° and 90°.

25. (d) cot 70°= cot (90° – 20°) = tan 20°
cot 80°= cot (90° – 10°) = tan 10°
so,
cot 10° . cot 80° . cot 20° . cot 70° . cot 60°
= (cot 10° . tan 10°) . (cot 20° . tan 20°) . cot 60°

$=1.1.\frac{1}{\sqrt{3}}=\frac{1}{\sqrt{3}}$

26. (a) $\sin\alpha\sec(30°+\alpha)=1$, $\frac{\sin\alpha}{\cos(30°+\alpha)}=1$

$\sin\alpha=\cos(30°+\alpha)=\sin(90°-30°+\alpha)$

$\sin\alpha=\sin(60°-\alpha)$

$\Rightarrow$ $\alpha=60°-\alpha$ or, $\alpha=30°$

$\sin\alpha+\cos 2\alpha=\sin 30°+\cos 60°=\frac{1}{2}+\frac{1}{2}=1$

27. (b) $\cos^2\theta+\cos^4\theta=1$, $1-\cos^2\theta=\cos^4\theta$
$\sin^2\theta=\cos^4\theta$, $\tan^2\theta=\cos^2\theta$
$\tan^2\theta+\tan^4\theta=\cos^2\theta+\cos^4\theta=1$

28. (b) tan 86°= tan (90° – 4°) = cot 4°
tan 43° = cot 47°, tan 4°·tan 43°·tan 47°·tan 86°
(tan 4°·tan 86°) (tan 43°·tan 47°)
(tan 4°·cot 4°) · (tan 47°·cot 47°) = 1 × 1 = 1

29. (d) $\sin 17°=\frac{x}{y}$

$\cos 17°=\sqrt{1-\frac{x^2}{y^2}}=\frac{\sqrt{y^2-x^2}}{y}$

$\sec 17°=\frac{y}{\sqrt{y^2-x^2}}$

sin 73°= sin (90° – 17°) = cos 17°

$\sin 73°=\frac{\sqrt{y^2-x^2}}{y}$

so, sec 17°– sin 73°

$=\frac{y}{\sqrt{y^2-x^2}}-\frac{\sqrt{y^2-x^2}}{y}=\frac{y^2-(y^2-x^2)}{y\sqrt{y^2-x^2}}$

$=\frac{x^2}{y\sqrt{y^2-x^2}}$

30. (b); $\frac{1}{\cos x}=\frac{1}{\sin y}$

$\cos y=\sqrt{1-\sin^2 y}=\sqrt{1-\cos^2 x}=\sqrt{\sin^2 x}=\sin x.$

sin (x + y)= sinx . cosy + cosx siny.
$=\sin^2 x+\cos^2 x=1.$

31. (a); tan 89° = tan (90° – 1°) = cot 1°
similarly, tan 88° = cot 2°;
tan 87° = cot 3°, ...
tan 1° tan 2° tan 3° .... tan 89° =
(tan 1° · tan 2° · tan 3° ....tan 45° .... tan 87° .
tan 88° · tan 89°·
= (tan 1°· tan 89°) · (tan 2° · tan 88°) · (tan 3° · tan 87°) .... tan 45°.
= (tan 1° · cot 1°) · (tan 2° · cot 2°) · (tan 3° · cot 3°)
= 1

32. (c) $A+B+C=\pi$

$\frac{A+B}{2}=\frac{\pi-C}{2}=\frac{\pi}{2}-\frac{C}{2}$

$\sin\left(\frac{A+B}{2}\right)=\sin\left(\frac{\pi}{2}-\frac{C}{2}\right)$

$=\sin\left(\frac{A+B}{2}\right)=\cos\frac{C}{2}$

$\cos\left(\frac{A+B}{2}\right)=\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)=\sin\frac{C}{2}$

$\tan\left(\frac{A+B}{2}\right)=\tan\left(\frac{\pi}{2}-\frac{C}{2}\right)=\cot\frac{C}{2}\neq\sec\frac{C}{2}$

$\cot\left(\frac{A+B}{2}\right)=\cot\left(\frac{\pi}{2}-\frac{C}{2}\right)=\tan\frac{C}{2}$

33. (c) $(\sec A - \cos A)^2 + (\operatorname{cosec} A - \sin A)^2 - (\cot A - \tan A)^2$
$= \sec^2 A + \cos^2 A - 2\sec A \cdot \cos A + \operatorname{cosec}^2 A + \sin^2 A - 2\operatorname{cosec} A \cdot \sin A - [\cot^2 A + \tan^2 A - 2\cot A \cdot \tan A]$
$= (\sec^2 A - \tan^2 A) + (\cos^2 A + \sin^2 A) + (\operatorname{cosec}^2 A - \cot^2 A) - 2 - 2 + 2$
$= 1 + 1 + 1 - 2 = 1$

34. (d) $7\sin^2\theta + 3\cos^2\theta = 4$
$4\sin^2\theta + 3\sin^2\theta + 3\cos^2\theta = 4$
$4\sin^2\theta + 3 = 4, \quad 4\sin^2\theta = 1$
$\sin\theta = \frac{1}{2}$ [$\because$ θ in an acute angle]
$\Rightarrow \operatorname{cosec}\theta = \frac{1}{\sin\theta} = 2$

35. (a) $\cot 18° \cot 72° \cos^2 22° + \frac{\cot 18°}{\tan 72° \sec^2 68°}$
$= \cot 18° \cdot \tan 18° \cdot \cos^2 22° + \left(\frac{\cot 18°}{\cot 18°}\right) \times \cos^2 68°$
$= \cos^2 22° + \cos^2 68°, \quad = \cos^2 22° + \sin^2 22° = 1$

36. (c) $\frac{1}{\operatorname{cosec}^2 51°} + \sin^2 39° + \tan^2 51° - \frac{1}{\sin^2 51° \cdot \sec^2 39°}$
$= \sin^2 51° + \sin^2 39° + \tan^2 (90° - 39°) - \frac{1}{\sin^2(90° - 39°) \cdot \sec^2 39°}$
$= \cos^2 39° + \sin^2 39° + \cot^2 39° - \frac{1}{\cos^2 39° \cdot \sec^2 39°}$
$= 1 + \cot^2 39° - 1, \quad = \operatorname{cosec}^2 39° - 1 = x^2 - 1$

37. (b) By componendo and dividendo :
$\frac{\tan\theta + \cot\theta}{\tan\theta - \cot\theta} = \frac{\tan\theta + \cot\theta + \tan\theta - \cot\theta}{\tan\theta + \cot\theta - \tan\theta + \cot\theta} = \frac{2+1}{2-1}$
$\frac{2\tan\theta}{2\cot\theta} = 3, \quad \frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\cos\theta} = 3$
or, $\sin^2\theta = 3\cos^2\theta, \quad \sin^2\theta = 3(1 - \sin^2\theta)$
$4\sin^2\theta = 3$
$\sin\theta = \frac{\sqrt{3}}{2}$ [as $0° \le \theta \le 90°$]

38. (d) $\sin\theta + \operatorname{cosec}\theta = 2, \quad \sin\theta + \frac{1}{\sin\theta} = 2$
$\sin^2\theta - 2\sin\theta + 1 = 0, \quad (\sin\theta - 1)^2 = 0, \sin\theta = 1$
so, $\sin^5\theta + \operatorname{cosec}^5\theta = \sin^5\theta + \frac{1}{\sin^5\theta}, = 1 + 1 = 2$

39. (b); $\cos^2 5° + \cos^2 10° + \cos^2 15° + ...+ \cos^2 80° + \cos^2 85° + \cos^2 90°$
$= (\cos^2 5° + \cos^2 85°) + (\cos^2 10° + \cos^2 80°) + ... \cos^2 40° + \cos^2 50°) + \cos^2 45° + \cos^2 90°$
$= 1 + 1 + ...$ 8 times $+ \frac{1}{2} + 0 = 8\frac{1}{2}$

40. (c); $2\cos\theta - \sin\theta = \frac{1}{\sqrt{2}}$
Let $2\sin\theta + \cos\theta = x$
on squaring and adding :
$4\cos^2\theta + \sin^2\theta - 4\sin\theta\cos\theta + 4\sin^2\theta + \cos^2\theta + 4\sin\theta \cdot \cos\theta = \frac{1}{2} + x^2$
$4(\cos^2\theta + \sin^2\theta) + (\cos^2\theta + \sin^2\theta) = \frac{1}{2} + x^2$
$4 + 1 = \frac{1}{2} + x^2, \quad x^2 = 5 - \frac{1}{2}, x^2 = \frac{9}{2} = \frac{3}{\sqrt{2}}$

41. (c); $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$
$4\cos\theta = 2\sin\theta, \quad \tan\theta = 2, \quad \sin^4\theta - \cos^4\theta,$
$(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$
$\Rightarrow \cos^2\theta(\tan^2\theta - 1)$
$\Rightarrow \frac{3}{\sec^2\theta} \Rightarrow \frac{3}{1 + \tan^2\theta} = \frac{3}{1+4} = \frac{3}{5}$

42. (d) $\frac{\sin 39°}{\cos 51°} + 2\tan 11° \cdot \tan 79°$
$\tan 31° \cdot \tan 59° \tan 45° - 3(\sin^2 21° + \sin^2 69°)$
$= \frac{\sin 39°}{\cos(90° - 39°)} + 2\tan 11° \times \tan(90° - 11°)$. tan $31° \cdot \tan(90° - 31°) \cdot 1 - 3[\sin^2 21° + \sin^2(90° - 21°)]$
$= \frac{\sin 39°}{\sin 39°} + 2\tan 11° \cdot \cot 11° \cdot \tan 31° \cdot \cot 31° - 3(\sin^2 21° + \cos^2 21°) = 1 + 2 - 3 = 0$

43. (c) $A = \tan 11° \times \tan 29°$,
$B = 2\cot 61° \cot 79°$
$= 2\cot(90° - 29°)\cot(90° - 11°)$
$= 2\tan 29° \times \tan 11° = 2A$

44. (b) $\cos^2\alpha + \cos^2\beta = 2$
$1 - \sin^2\alpha + 1 - \sin^2\beta = 2$
$\sin^2\alpha + \sin^2\beta = 0, \quad \sin\alpha = \sin\beta = 0$
$\alpha = \beta = 0, \quad \tan^3\alpha + \sin^5\beta = 0$

45. (a) $\sin\theta - \cos\theta = \frac{7}{13}$ ...(i)
Let $\sin\theta + \cos\theta = x$ ...(ii)
on squaring both equations and adding :

$2(\sin^2\theta + \cos^2\theta) = \frac{49}{169} + x^2$

$x^2 = 2 - \frac{49}{169} = \frac{338 - 49}{169}$, $x^2 = \frac{289}{169}$, $x = \frac{17}{13}$

46. (d) $\sin(2x - 20°) = \cos(2y + 20°)$
$\sin(2x - 20°) = \sin(90° - 2y - 20°)$
$2x - 20° = 70° - 2y$
$2x + 2y = 90°$
$x + y = 45°$
$\tan(x + y) = \tan 45° = 1$.

47. (d) $(\sec x . \sec y + \tan x . \tan y)^2 - (\sec x \tan y + \tan x . \sec y)^2$
$= \sec^2 x . \sec^2 y + \tan^2 x . \tan^2 y + 2 \sec x . \sec y . \tan x . \tan y - \sec^2 x \tan^2 y - \tan^2 x . \sec^2 y - 2 \sec x \tan y . \tan x . \sec y$
$= \sec^2 x . \sec^2 y + \tan^2 x . \tan^2 y - \sec^2 x \tan^2 y - \tan^2 x . \sec^2 y$
$= \sec^2 x (\sec^2 y - \tan^2 y) - \tan^2 x (\sec^2 y - \tan^2 y)$
$= \sec^2 x - \tan^2 x = 1$

48. (b) x^{th} term,
$a_n = a + (n-1)d$, $85 = 5 + (n-1)1$
$80 = n - 1$ $n = 81$
$\Rightarrow$ 80 terms + 1 middle term.
So, $\sin^2 5° + \sin^2 6° + \ldots + \sin^2 45° + \ldots + \sin^2 84° + \sin^2 85°$
$= (\sin^2 5° + \sin^2 85°) + \ldots + (\sin^2 6° + \sin^2 84°) + \ldots$ upto ... to 40 terms $+ \sin^2 45°$
$= (\sin^2 5° + \cos^2 5°) + (\sin^2 6° + \cos^2 6°) + \ldots$ to 40 terms $+ \sin^2 45°$
$= 40 + \frac{1}{2} = 40\frac{1}{2}$

49. (b) $\sin\theta + \operatorname{cosec}\theta = 2$, $\sin\theta + \frac{1}{\sin\theta} = 2$
$(\sin\theta - 1)^2 = 0$, $\sin\theta - 1 = 0$
$\sin\theta = 1 \Rightarrow \operatorname{cosec}\theta \Rightarrow = 1$
So, $\sin^{100}\theta + \operatorname{cosec}^{100}\theta$, $= 1 + 1 = 2$

50. (b) $\tan 7° . \tan 23° \tan 60° . \tan 67° . \tan 83°$
$= \tan 7° . \tan 83° . \tan 23° . \tan 67° . \tan 60°$
$= (\tan 7° . \cot 7°) . (\tan 23° . \cot 23°) . \tan 60°$
$= 1.1.\sqrt{3} = \sqrt{3}$.

Relation Between Trigonometric Ratios

S.no	Identity	Relation
1	tan A	sin A/cos A
2	cot A	cos A/sin A
3	cosec A	1/sin A
4	sec A	1/cos A