Chapter - 5

ALGEBRA

Foundation

Solutions

1. (a); $5 * 4 = (5+3)^2 (4-1) = (8^2)(3)$
$= 64 \times 3 = 192$

2. (a); $a \times b = a^b$
$\therefore \quad 5 \times 3 = 5^3 = 125$

3. (a); $1.5a = 0.04b$
$$\frac{b}{a} = \frac{1.5}{0.04} = \frac{150}{4}$$
By componendo and dividendo
$$\frac{b-a}{b+a} = \frac{150-4}{150+4} = \frac{146}{154} = \frac{73}{77}$$

4. (a); $$\frac{a^2+b^2+ab}{a^3-b^3} = \frac{a^2+b^2+ab}{(a-b)(a^2+b^2+ab)}$$
$$= \frac{1}{a-b} = \frac{1}{11-9} = \frac{1}{2}$$

5. (a); $a^4 + b^4 - a^2b^2 = 0$ ...(i)
Now,
$a^6 + b^6 = (a^2)^3 + (b^2)^3$
$= (a^2 + b^2)(a^4 + b^4 - a^2b^2)$
From equation (i)
$a^6 + b^6 = 0$

6. (c); $\frac{x}{y} = 2 \Rightarrow x = 2y$
$x - 2y = 0$ ...(i)
$5x^2 - 13xy + 6y^2$
$= 5x^2 - 10xy - 3xy + 6y^2$
$= 5x(x-2y) - 3y(x-2y)$
$= (x-2y)(5x-3y) = 0 \times (5x-3y) = 0$

7. (b); $(a-b)^2 = a^2 - 2ab + b^2$
$x^4 - 2x^2 + K = (x^2)^2 - 2 \times x^2 \times 1 + K$
$\therefore \quad K = (1)^2 = 1$

8. (a); Here, $a = 1.21$, $b = 2.12$ and $c = -3.33$
$a + b + c = 1.21 + 2.12 - 3.33$
$= 3.33 - 3.33 = 0$
$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$
Hence, $a^3 + b^3 + c^3 - 3abc = 0$

9. (b); $x * y = x^2 + y^2 - xy$
$9 * 11 = 9^2 + 11^2 - 9 \times 11$
$= 81 + 121 - 99 = 202 - 99 = 103$

10. (a); $p = 999$ (Given)
Now, $\sqrt[3]{p(p^2+3p+3)+1}$
$= \sqrt[3]{p^3+3p^2+3p+1} = \sqrt[3]{(p+1)^3}$
$= p + 1 = 999 + 1 = 1000$

11. (b); $X * Y = X^2 + Y^2 - XY$
$11 * 13 = 11^2 + 13^2 - 11 \times 13$
$= 121 + 169 - 143 = 290 - 143 = 147$

12. (b); $a^3b = abc \Rightarrow a^2 = c$
$\therefore \quad a^3b = abc = 180$
$= 1^2 \times 180 \times 1 = 1^3 \times 180$
$\Rightarrow \quad c = 1$

13. (c); $$\frac{1}{x^{99}} + \frac{1}{x^{98}} + \frac{1}{x^{97}} + \frac{1}{x^{96}} + \frac{1}{x^{95}} + \frac{1}{x^{94}} + \frac{1}{x} - 1$$
Put $x = -1$
$$= \frac{1}{(-1)^{99}} + \frac{1}{(-1)^{98}} + \frac{1}{(-1)^{97}} + \frac{1}{(-1)^{96}} + \frac{1}{(-1)^{95}} + \frac{1}{(-1)^{94}} + \frac{1}{(-1)} - 1$$
$= -1 + 1 - 1 + 1 - 1 + 1 - 1 - 1 = -2$

14. (c); Given, $x = 6$
$$\frac{4 \times 6}{3} + 2P = 12$$
$\Rightarrow \quad 8 + 2P = 12$
$2P = 4$, $\quad P = 2$

15. (b); $a^{1/3} = 11 \quad \Rightarrow \quad a = 11^3 = 1331$
$\therefore \quad a^2 - 331a = a(a - 331)$
$= 1331(1331 - 331)$
$= 1331 \times 1000 = 1331000$

16. (b); $x + \frac{1}{x} = 2$
$x = 1$, satisfies the above equation
$\therefore \quad$ put $x = 1$ in $x^2 + \frac{1}{x^3} = (1)^2 + \frac{1}{(1)^3} = 2$

17. (b); $x^2 - 3x + 1 = 0$

$x^2 + 1 = 3x$

Divide whole equation by x

$x + \frac{1}{x} = 3$, $x^3 + \frac{1}{x^3} = (3)^3 - 3(3) = 27 - 9 = 18$

18. (c); $m + \frac{1}{m-2} = 4$

$(m-2) + \frac{1}{(m-2)} = 4 - 2$, $(m-2) + \frac{1}{(m-2)} = 2$

Here, $(m - 2) = 1$

$\Rightarrow (m-2)^2 + \frac{1}{(m-2)^2} = 1^2 + \frac{1}{1^2} = 2$

19. (b); $a + \frac{1}{a} = \sqrt{3}$

$a^3 + \frac{1}{a^3} = \left(\sqrt{3}\right)^3 - 3\sqrt{3} = 3\sqrt{3} - 3\sqrt{3} = 0$

20. (b); $\frac{a}{b} + \frac{b}{a} - 1 = 0$

$a^2 + b^2 - ab = 0$

$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$= (a + b) \times 0$

$a^3 + b^3 = 0$

21. (c); Here $x = 3$, $y = 5$, $z = 4$

$\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{16} = \frac{9}{9} + \frac{25}{25} + \frac{16}{16} = 1 + 1 + 1 = 3$

22. (a); $\frac{a}{b} = \frac{2}{3}$ and $\frac{b}{c} = \frac{4}{5}$

$\frac{a}{b} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$, $\frac{b}{c} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$

Required ratio,

$\frac{a+b}{b+c} = \frac{8+12}{12+15} = \frac{20}{27}$

23. (a); $\frac{y}{x} = \frac{4}{15}, \frac{x}{y} = \frac{15}{4}$

By componendo and dividendo

$\frac{x-y}{x+y} = \frac{15-4}{15+4} = \frac{11}{19}$

24. (b); Given $\frac{3x+2y}{3x-2y} = \frac{4}{3}$

$\Rightarrow 9x + 6y = 12x - 8y$

$14y = 3x$

$= \frac{\frac{x^2}{y^2}+1}{\frac{x^2}{y^2}-1} = \frac{\left(\frac{14}{3}\right)^2+1}{\left(\frac{14}{3}\right)^2-1} = \frac{205}{187}$

25. (d); Given, $x + \frac{1}{x} = 5$

$\Rightarrow \frac{3x}{2x^2 + 2 - 5x} = \frac{3x}{x\left[\left(2x + \frac{2}{x}\right) - 5\right]}$

$\Rightarrow \frac{3}{2\left(x + \frac{1}{x}\right) - 5} = \frac{3}{10-5} = \frac{3}{5}$

26. (c); $\left(a + \frac{1}{a}\right)^2 = 3$

$a + \frac{1}{a} = \sqrt{3}$, $a^3 + \frac{1}{a^3} = 3\sqrt{3} - 3\sqrt{3}$

$a^3 + \frac{1}{a^3} = 0$

$\therefore a^3 + \frac{1}{a^3} + 3\sqrt{3} = 0 + 3\sqrt{3} = 3\sqrt{3}$

27. (a); $P = \frac{x^2 - 36}{x^2 - 49}$

$= \frac{(x+6)(x-6)}{(x+7)(x-7)}$

$Q = \frac{x+6}{x+7}$, $P = \frac{Q \times (x-6)}{(x-7)}$, $\frac{P}{Q} = \frac{x-6}{x-7}$

28. (d); If $x + \frac{1}{x} = 2$

then $x = 1$

$x^{17} + \frac{1}{x^{19}} = (1)^{17} + \frac{1}{(1)^{19}} = 1 + 1 = 2$

29. (b); Given $\frac{2x^4 - 162}{(x^2+9)(2x-6)}$

$= \frac{2(x^4 - 81)}{2(x^2+9)(x-3)} = \frac{2(x^2-9)(x^2+9)}{2(x^2+9)(x-3)}$

$= \frac{x^2-9}{x-3} = \frac{(x+3)(x-3)}{x-3} = x + 3$

30. (b); Given, $\frac{a^2-b^2-2bc-c^2}{a^2+b^2+2ab-c^2}=\frac{a^2-(b+c)^2}{(a+b)^2-c^2}$

$=\frac{[a+(b+c)][a-(b+c)]}{(a+b+c)(a+b-c)}=\frac{a-b-c}{a+b-c}$

31. (a); Given,

$\frac{x+\frac{1}{x}}{2}=1,\ x+\frac{1}{x}=2$

then $x=1$,

$\therefore\ 8(1)^{10}+\frac{4}{(1)^5}=8+4=12$

32. (a); $x+y+z=0$

$\Rightarrow\ x+y=-z,\ y+z=-x,\ x+z=-y$

$\therefore\ (x+y)(y+z)(z+x)=(-z)(-x)(-y)=-xyz$

33. (c); $x^2-4x+1=0$

$x^2+1=4x$

Dividing the equation by x

$x+\frac{1}{x}=4$

$x^3+\frac{1}{x^3}=a^3-3a$

$=4^3-3(4)=64-12=52$

34. (a); Given $x+y+z=6$ and $xy+yz+zx=11$

$x^3+y^3+z^3-3xyz$

$=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$=(x+y+z)[(x+y+z)^2-3(xy+yz+zx)]$

$=6[6^2-3(11)]=6\times 3=18$

35. (a); $a^3+b^3+c^3-3abc$

$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$=\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$

$=\frac{1}{2}(258+260+262)[(-2)^2+(-2)^2+4^2]$

$=\frac{1}{2}\times 780\times 24=9360$

36. (b); Given, $pq+qr+rp=0$

$\Rightarrow -qr=pq+rp$

$\therefore\ \frac{p^2}{p^2-qr}+\frac{q^2}{q^2-rp}+\frac{r^2}{r^2-pq}$

$=\frac{p^2}{p^2+rp+pq}+\frac{q^2}{q^2+pq+qr}+\frac{r^2}{r^2+qr+rp}$

$=\frac{p}{p+q+r}+\frac{q}{p+q+r}+\frac{r}{p+q+r}=\frac{p+q+r}{p+q+r}=1$

37. (c); Given,

$u^3+(-2v)^3+(-3w)^3=3\times(-2)(-3)\ uvw$

$\therefore\ u+(-2v)+(-3w)=0$

$u-2v-3w=0$

$u-2v=3w$

38. (c); $(x^2+y^2)(p^2+q^2)=(xp+yp)^2$

$x^2p^2+x^2q^2+y^2p^2+y^2q^2=x^2p^2+y^2p^2+2xpyq$

$x^2q^2+y^2p^2-2xpyq=0$

$(xq-yp)^2=0$

$xq=yp$

39. (b); $(ab+bc+ca)^2$

$=a^2b^2+b^2c^2+c^2a^2+2abbc+2abca+2bcca$

Or, $0=a^2b^2+b^2c^2+c^2a^2+2abc(b+a+c)$

$0=a^2b^2+b^2c^2+c^2a^2+2pq$

$=a^2b^2+b^2c^2+c^2a^2=-2pq$

40. (b); $\frac{x}{y}=\frac{(p+q)^2}{(p-q)^2}$

By componendo and dividendo

$\frac{x-y}{x+y}=\frac{(p+q)^2-(p-q)^2}{(p+q)^2+(p-q)^2}=\frac{4pq}{2(p^2+q^2)}$

$=\frac{2pq}{p^2+q^2}$

Moderate

1. (a); Here, $\frac{a}{b}+\frac{b}{a}=1$

$\frac{a^2+b^2}{ab}=1,\ a^2+b^2=ab$

$a^2 + b^2 - ab = 0$

$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$a^3 + b^3 = (a + b) \times 0$

$a^3 + b^3 = 0, a^3 + b^3 + 2 = 2$

2. (a); $x^2+y^2+\frac{1}{x^2}+\frac{1}{y^2}=4$

Here x = y = 1, satisfies the above equation then $x^2 + y^2 = 1^2 + 1^2 = 2$

3. (b); $x^2 = y + z$

$\Rightarrow \quad x^2 + x = x + y + z$

$\Rightarrow \quad x(x + 1) = x + y + z$ ...(i)

Similarly,

$y(y + 1) = x + y + z$ ...(ii)

and

$z(z + 1) = x + y + z$ ...(iii)

On adding (i), (ii), and (iii)

$\therefore \quad \frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}$

$=\frac{x}{x+y+z}+\frac{y}{x+y+z}+\frac{z}{x+y+z}=\frac{x+y+z}{x+y+z}=1$

4. (a); $x+\frac{1}{x}=\sqrt{3}$

Cubing both sides

$x^3+\frac{1}{x^3}=3\sqrt{3}-3\sqrt{3}$

$x^6 + 1 = 0$

Now, $x^{18} + x^{12} + x^6 + 1$

$= x^{12}(x^6 + 1) + (x^6 + 1) = x^{12} \times 0 + 0 = 0$

5. (d); $(ad - bc)^2 + (ac + bd)^2$

$= a^2d^2 + b^2c^2 - 2acbd + a^2c^2 + b^2d^2 + 2abcd$

$= a^2d^2 + b^2c^2 + a^2c^2 + b^2d^2$

$= d^2(a^2 + b^2) + c^2(a^2 + b^2)$

$= (c^2 + d^2)(a^2 + b^2) = 1 \times 2 = 2$

6. (a); $x+\frac{2}{3+\frac{4}{5+\frac{7}{6}}}=10$

$x+\frac{2}{3+\frac{4\times 6}{30+7}}=10,\ x+\frac{2}{3+\frac{24}{37}}=10$

$x+\frac{2\times 37}{111+24}=10,\ x+\frac{2\times 37}{135}=10$

$x+\frac{74}{135}=10,\ x=10-\frac{74}{135}$

$x=\frac{1350-74}{135}=\frac{1276}{135}$

7. (d); $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=3$

$\Rightarrow \ a = 3b,\ \ c = 3d,\ \ e = 3f$

$\therefore \ \frac{2a^2+3c^2+4e^2}{2b^2+3d^2+4f^2}$

$=\frac{2\times 9b^2+3\times 9d^2+4\times 9f^2}{2b^2+3d^2+4f^2}$

$=\frac{9\left(2b^2+3d^2+4f^2\right)}{2b^2+3d^2+4f^2}=9$

8. (a); Given $a+\frac{1}{a}=6$

$\therefore \ a^4+\frac{1}{a^4}=\left(a^2+\frac{1}{a^2}\right)^2-2$

$=\left[\left(a+\frac{1}{a}\right)^2-2\right]^2-2$

$= [6^2 - 2]^2 - 2 = (34)^2 - 2 = 1154$

9. (d); $x^4+\frac{1}{x^4}=23$

Or, $\left(x^2+\frac{1}{x^2}\right)^2=23+2$

$x^2+\frac{1}{x^2}=\sqrt{25}=5$

$\left(x-\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}-2,\ =5-2$

$\left(x-\frac{1}{x}\right)^2=3$

10. (c); $x^3 + y^3 = 35$ and $x + y = 5$

$\Rightarrow \quad (x + y)(x^2 + y^2 - xy) = 35$

$x^2 + y^2 - xy = 7 \quad \text{...(i)}$

and, $(x + y)^2 = 25$

$x^2 + y^2 + 2xy = 25 \quad \text{...(ii)}$

On solving equation (i) and (ii)

$3xy = 18$

$xy = 6 \quad \text{...(iii)}$

$\Rightarrow \quad x + y = 5 \quad \text{...(iv)}$

from (iii) and (iv)

$$\frac{x+y}{xy} = \frac{5}{6}, \quad \frac{1}{x} + \frac{1}{y} = \frac{5}{6}$$

11. (c); $x^2 + y^2 + z^2 = 2x - 2y - 2z - 3$

$x^2 + y^2 + z^2 - 2x + 2y + 2z + 1 + 1 + 1 = 0$

$(x^2 - 2x + 1) + (y^2 + 2y + 1) + (z^2 + 2z + 1) = 0$

$(x - 1)^2 + (y + 1)^2 + (z + 1)^2 = 0$

$\therefore x = 1, \quad y = -1 \quad \text{and} \quad z = -1$

$5(1) - 4(-1) + 2(-1)$

$5 + 4 - 2, \quad 9 - 2 = 7$

12. (b); $2p + \frac{1}{p} = 4$

$$\Rightarrow p + \frac{1}{2p} = 2$$

$$\left(p + \frac{1}{2p}\right)^3 = p^3 + \frac{1}{8p^3} + 3(p)\frac{1}{2p}\left(p + \frac{1}{2p}\right)$$

$$2^3 = p^3 + \frac{1}{8p^3} + \frac{3}{2} \times (2), \quad p^3 + \frac{1}{8p^3} = 8 - 3 = 5$$

13. (b); $x = 3 + \sqrt{8}$ and

$$\frac{1}{x} = \frac{1}{3+\sqrt{8}} \times \frac{3-\sqrt{8}}{3-\sqrt{8}} = \frac{3-\sqrt{8}}{9-8} = 3 - \sqrt{8}$$

$$x + \frac{1}{x} = 3 + \sqrt{8} + 3 - \sqrt{8}, \quad x + \frac{1}{x} = 6$$

$$x^2 + \frac{1}{x^2} = 6^2 - 2 = 36 - 2 = 34$$

14. (c); Given

$$\frac{2a+b}{a+4b} = 3$$

$2a + b = 3a + 12b$

$a = -11b$

Then,

$$\frac{a+b}{a+2b} = \frac{-11b+b}{-11b+2b} = \frac{-10b}{-9b} = \frac{10}{9}$$

15. (c); Here, $a = 4.965$, $b = 2.343$, $c = 2.622$

$a - b - c = 4.965 - 2.343 - 2.622$

$= 4.965 - 4.965$

$a - b - c = 0$

When $a - b - c = 0$ then

$a^3 - b^3 - c^3 - 3abc = 0$

16. (b); $x * y = 3x + 2y$,

$2 * 3 + 3 * 4 = 3(2) + 2(3) + 3(3) + 2(4)$

$= 6 + 6 + 9 + 8 = 29$

17. (b); $x^4 - 17x^3 + 17x^2 - 17x + 17$

$= 16x^3 - 16x^3 - x^3 + 16x^2 + x^2 - 16x - x + 17$

$= 16x^3 - 16x^3 - 16x^2 + 16x^2 + 16x - 16x - 16 + 17$

$= 17 - 16 = 1$

18. (d); For maximum value

$$a = b = c = d = \frac{1}{4}$$

$(1 + a)(1 + b)(1 + c)(1 + d)$

$$= \left(1+\frac{1}{4}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{4}\right) = \left(\frac{5}{4}\right)^4$$

19. (a); $$\frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1} = a\sqrt[3]{4} + b\sqrt[3]{2} + c$$

$$\Rightarrow \frac{1}{2^{2/3} + 2^{1/3} + 1} = a(2)^{2/3} + b(2)^{1/3} + c$$

On multiplying numerator and denominator by $\left(2^{1/3} - 1\right)$

$$\Rightarrow \frac{\left(2^{1/3} - 1\right)}{\left(2^{1/3} - 1\right)\left(2^{2/3} + 2^{1/3} + 1\right)} = a \cdot 2^{2/3} + b \cdot 2^{1/3} + c$$

$$\frac{2^{1/3} - 1}{2 - 1} = a \cdot 2^{2/3} + b \cdot 2^{1/3} + c$$

$\therefore \quad a = 0, \ b = 1, \ c = -1$

$a + b + c = 0 + 1 - 1 = 0$

20. (a); $a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0$

$\Rightarrow \quad (a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$

$= 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$

$= 3(a - b)(a + b)(b - c)(b + c)(c - a)(c + a)$

$(a + b)(a - b)$ is the factor of given equation

21. (d); $p(p^2 + 3p + 3)$

$= p^3 + 3p^2 + 3p + 1 - 1$

$= (p + 1)^3 - 1 = (99 + 1)^3 - 1$

$= 100^3 - 1 = 1000000 - 1$

$= 999999$

22. (d); The expression is $\sqrt[3]{p\left(p^2+3p+3\right)+1}$

$= \sqrt[3]{p^3+3p^2+3p+1} = \left[\left(p+1\right)^3\right]^{1/3} = p+1$

= 123 + 1 = 124

23. (d); $(a-b)^2 = a^2 - 2ab + b^2$

$x^4 - 4x^2 + k = (x^2)^2 - 2(x^2) \times 2 + 4$

$k = 4$

24. (d); $x = 7-4\sqrt{3}$

$$\frac{1}{x} = \frac{1}{7-4\sqrt{3}} = \frac{1\left(7+4\sqrt{3}\right)}{\left(7+4\sqrt{3}\right)\left(7-4\sqrt{3}\right)}$$

$$= \frac{7+4\sqrt{3}}{49-48} = 7+4\sqrt{3}$$

$$x+\frac{1}{x} = 7-4\sqrt{3}+7+4\sqrt{3}, \quad x+\frac{1}{x} = 14$$

25. (a); $x = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$$= \frac{\left(\sqrt{3}+1\right)^2}{3-1} = \frac{3+1+2\sqrt{3}}{2}$$

$$= \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$$

Similarly, $y = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3}$

$$\therefore \quad x^2+y^2 = \left(2+\sqrt{3}\right)^2 + \left(2-\sqrt{3}\right)^2$$

$$= 2\left(2^2+\left(\sqrt{3}\right)^2\right) = 2(4+3) = 14$$

26. (b); $a = \frac{\sqrt{5}+1}{\sqrt{5}-1} = \frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$

$$= \frac{\left(\sqrt{5}+1\right)^2}{5-1} = \frac{5+1+2\sqrt{5}}{4}$$

$$a = \frac{3+\sqrt{5}}{2}$$

Similarly,

$$b = \frac{3-\sqrt{5}}{2}$$

$$a+b = \frac{3+\sqrt{5}}{2} + \frac{3-\sqrt{5}}{2} = 3$$

and $ab = \frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}-1}{\sqrt{5}+1} = 1$

$$\frac{a^2+ab+b^2}{a^2-ab+b^2} = \frac{(a+b)^2-ab}{(a+b)^2-3ab}$$

$$= \frac{3^2-1}{3^2-3} = \frac{9-1}{9-3} = \frac{8}{6} = \frac{4}{3}$$

27. (b); Given $x - \frac{1}{x} = 4$

$$\left(x+\frac{1}{x}\right)^2 = \left(x-\frac{1}{x}\right)^2 + 4 = 4^2 + 4 = 16 + 4$$

$$\left(x+\frac{1}{x}\right)^2 = 20$$

$$x+\frac{1}{x} = \sqrt{20} = \sqrt{2\times2\times5} = 2\sqrt{5}$$

28. (b); $x = 3+2\sqrt{2}$

$$\frac{1}{x} = \frac{1}{3+2\sqrt{2}} = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$\frac{1}{x} = 3-2\sqrt{2}, \quad x+\frac{1}{x} = 3+2\sqrt{2}+3-2\sqrt{2}$$

$$x+\frac{1}{x} = 6$$

$$\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 = x+\frac{1}{x}-2, \quad \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 = 6-2 = 4$$

$$\sqrt{x}-\frac{1}{\sqrt{x}} = 2$$

29. (c); $x+\frac{1}{2x} = 2$

Multiplying the above equation by 2

$$2x+\frac{1}{x} = 4$$

On cubing both sides

$$\left(2x+\frac{1}{x}\right)^3 = (2x)^3 + \left(\frac{1}{x}\right)^3 + 3(2x)\left(\frac{1}{x}\right)\left(2x+\frac{1}{x}\right)$$

$$64 = 8x^3 + \frac{1}{x^3} + 6\times4$$

$$8x^3 + \frac{1}{x^3} = 64 - 24 = 40$$

30. (a); $\left(x+\frac{1}{x}\right)^2=3,\ x+\frac{1}{x}=\sqrt{3}$

$x^3+\frac{1}{x^3}=\left(\sqrt{3}\right)^3-3\sqrt{3}$

$x^3+\frac{1}{x^3}=3\sqrt{3}-3\sqrt{3}$

$x^6+1=0$

$x^{72}+x^{66}+x^{54}+x^{48}+x^{36}+x^{30}+x^{24}+x^{18}+x^6+1$

$=x^{66}(x^6+1)+x^{48}(x^6+1)+x^{30}(x^6+1)+x^{18}(x^6+1)$

$+x^6+1$

$=x^{66}\times 0+x^{48}\times 0+x^{30}\times 0+x^{18}\times 0+0=0$

31. (b); $n=7+4\sqrt{3}=7+2\times 2\times\sqrt{3}$

$n=4+3+2\times 2\sqrt{3}=\left(2+\sqrt{3}\right)^2,\quad \sqrt{n}=2+\sqrt{3}$

$\frac{1}{\sqrt{n}}=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}},\quad \frac{1}{\sqrt{n}}=2-\sqrt{3}$

$\therefore\quad \sqrt{n}+\frac{1}{\sqrt{n}}=2+\sqrt{3}+2-\sqrt{3}=4$

32. (a); $\frac{x}{b+c}=\frac{y}{c+a}=\frac{x-y}{b+c-c-a}=\frac{x-y}{b-a}$

$\frac{y}{c+a}=\frac{z}{b+a}=\frac{y-z}{c+a-b-a}=\frac{y-z}{c-b}$

$\frac{z}{b+a}=\frac{x}{b+c}=\frac{z-x}{b+a-b-c}=\frac{z-x}{a-c}$

$\therefore\quad \frac{x-y}{b-a}=\frac{y-z}{c-b}=\frac{z-x}{a-c}$

33. (b); $x^2+y^2+z^2=2x-2y-2z-3$

$x^2+y^2+z^2-2x+2y+2z+1+1+1=0$

$(x^2-2x+1)+(y^2+2y+1)+(z^2+2z+1)=0$

$(x-1)^2+(y+1)^2+(z+1)^2=0$

$\therefore\quad x=1,\quad y=-1\quad$ and $\quad z=-1$

$2x-3y+4z=2+3-4=1$

34. (c); $x+y+z=6$

On Squaring,

$(x+y+z)^2=36$

$x^2+y^2+z^2+2xy+2yz+2xz=36$

$20+2(xy+yz+zx)=36$

$2(xy+yz+zx)=16$

$xy+yz+zx=8$

$\therefore\ x^3+y^3+z^3-3xyz$

$=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$=6(20-8)=6\times 12=72$

35. (c); $x+\frac{1}{x}=99$

$\therefore\quad \frac{100x}{2x^2+102x+2}=\frac{100x}{2x^2+2+102x}$

On dividing by x,

$=\frac{100}{2x+\frac{2}{x}+102}=\frac{100}{2\left(x+\frac{1}{x}\right)+102}$

$=\frac{100}{2\times 99+102}=\frac{100}{300}=\frac{1}{3}$

36. (b); $x\left(3-\frac{2}{x}\right)=\frac{3}{x}$

$3x-2=\frac{3}{x}\Rightarrow 3x-\frac{3}{x}=2$

$\Rightarrow\quad x-\frac{1}{x}=\frac{2}{3}$

On squaring both sides

$\left(x-\frac{1}{x}\right)^2=\frac{4}{9}\Rightarrow x^2+\frac{1}{x^2}-2=\frac{4}{9}$

$x^2+\frac{1}{x^2}=\frac{4}{9}+2=2\frac{4}{9}$

37. (a); $x^2-3x+1=0$

$x+\frac{1}{x}=3$

On squaring both sides

$\left(x+\frac{1}{x}\right)^2=9$

$x^2+\frac{1}{x^2}+2=9,\quad x^2+\frac{1}{x^2}=7$

$\Rightarrow\quad x^2+x+\frac{1}{x}+\frac{1}{x^2}$

$=\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)$

$=7+3=10$

38. (d); $x-a=\frac{1}{x-b}$

$\Rightarrow\quad (x-a)^3-\frac{1}{(x-a)^3}$

$$= \left(x - a - \frac{1}{x-a}\right)^3 + 3\left(x - a - \frac{1}{x-a}\right)$$

$= (x – a – x + b)^3 + 3 (x – a – x + b)$

$= (b – a)^3 + 3 (b – a)$

$= 5^3 + 3 \times 5 = 125 + 15 = 140$

39. (c); $\frac{x}{y} = \frac{3}{2}$

$$\frac{x^2}{y^2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}, \quad \frac{2x^2 + 3y^2}{3x^2 - 2y^2} = \frac{2\left(\frac{x^2}{y^2}\right) + 3}{3\left(\frac{x^2}{y^2}\right) - 2}$$

On dividing numerator and denominator by y^2.

Now,

$$= \frac{\left(2 \times \frac{9}{4}\right) + 3}{\left(3 \times \frac{9}{4}\right) - 2} = \frac{\frac{9}{2} + 3}{\frac{27}{4} - 2}$$

$$= \frac{\frac{9+6}{2}}{\frac{27-8}{4}} = \frac{15}{2} \times \frac{4}{19} = \frac{30}{19} = 30 : 19$$

40. (d); Given, $D : C = 6 : 5$

$C : B = 4 : 3$ and $B : A = 2 : 1$

$\therefore \quad D : C : B : A$

$= 6 \times 4 \times 2 : 5 \times 4 \times 2 : 5 \times 3 \times 2 : 5 \times 3 \times 1$

$= 48 : 40 : 30 : 15$

Algebra Formulas including Natural Numbers

→ **Algebraic formulas involving natural numbers can include equations and expressions such as:**

1. **Addition:** a + b = c
2. **Subtraction:** a – b = c
3. **Multiplication:** a × b = c
4. **Division: a/b =** c, where b ≠ 0.
5. **Exponentiation: an =** c, where n is a natural number.
6. **Square of a number:** a2 = c
7. **Cube of a number:** a^3 = c
8. **Square root: √a =** c, where c is the non-negative square root of a.
9. **Cube root: ∛a =** c, where c is the real root of a

Algebra Formulas (Law of Exponents)

→ The law of exponents in algebra are rules that govern the manipulation and simplification of expressions involving exponents. **Here are the basic laws:**

1. **Product Rule: am × an = am + n**
2. **Quotient Rule: am÷ an = am – n**
3. **Power Rule: (am)n = amn**
4. **Zero Exponent Rule: a0 = 1, (Where a ≠ 0)**
5. **Negative Exponent Rule: a-1 = 1/a1, (where a ≠ 0)**