Chapter - 4

Co-ordinate Geometry

Foundation

Solutions

1. (a); a + 2 = 3a + 5

$\Rightarrow \quad a = \frac{-3}{2}$

So, the given point is $\left(\frac{-3}{2}, \frac{1}{2}\right)$

$\Rightarrow$ The distance of the point from y-axis $= \frac{3}{2}$.

2. (b); mid-point of given points is:

$\left(\frac{-3+7}{2}, \frac{5-6}{2}\right) \equiv \left(2, \frac{-1}{2}\right)$

The distances of the above point from x and y axes are $\frac{1}{2}$ and 2 respectively.

difference $= 2 - \frac{1}{2} = \frac{3}{2}$

3. (c); 12x – 9y = 108

At $x = 0, \;\; y = -12$

At $y = 0, \;\; x = 9$

So, the given line meets the coordinate axes at (9, 0) and (0, – 12).

So, the required length $= \sqrt{(9-0)^2 + (0+12)^2}$

$= \sqrt{81 + 144} = 15$ units

4. (a); y = mx + c

At $x = 0, \;\; y = c$

At $y = 0, \;\; x = \frac{-c}{m}$

The given line touches the coordinate axes at (0, c) and $\left(\frac{-c}{m}, 0\right)$.

Required length $= \sqrt{\left(0 - \left(-\frac{c}{m}\right)\right)^2 + (c-0)^2}$

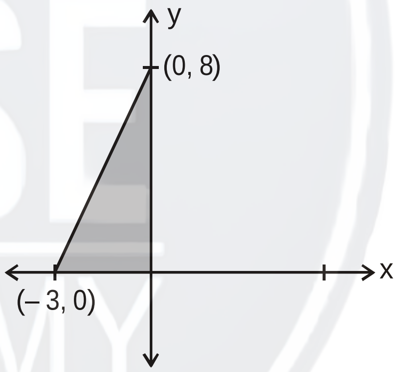
$= \sqrt{\frac{c^2}{m^2} + c^2} = \frac{c}{m}\sqrt{1+m^2}$

5. (b); 8x – 3y = – 24

At $x = 0, \;\; y = 8$

At $y = 0, \;\; x = -3$

So, the given line touches the coordinate axes at (0, 8) and (– 3, 0).



So, area of the triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 3 \times 8 = 12$ sq. units

6. (c); 4x – 3y = – 4 (i)

4x + 3y = 20 (ii)

Adding (i) and (ii), we get

8x = 16 $\Rightarrow$ x = 2

Put x = 2 in (ii)

$\Rightarrow$ y = 4

So, the given lines intersect at (2, 4).

Now, the given lines intersect the x-axis at (– 1, 0). and (5, 0).

So, base of the triangle = 5 – (– 1) = 6

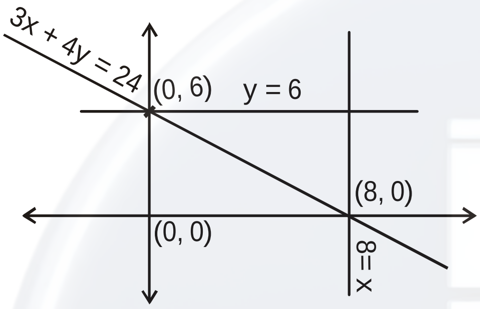
height = 4

Area $= \frac{1}{2} \times 6 \times 4 = 12$ sq. units

7. (a); $3x - 4y = 0$ ...(i)
$x = 4$ ...(ii)
The given lines intersect at (4, 3) and they intersect the x-axis at (0, 0) and (4, 0)
$\Rightarrow$ Base = 4 units
height = 3 units

$\text{Area} = \frac{1}{2} \times 4 \times 3 = 6$ sq. units

8. (b); $3x + 4y = 24$ ...(i)
$x = 8$ ...(ii)
$y = 6$ ...(iii)
From (i):
At $x = 0,\ y = 6$
At $y = 0,\ x = 8$
So, line (i) intersects the axes at (0, 6) and (8, 0).



Required area $= \frac{1}{2} \times 8 \times 6 = 24$ sq. units

9. (d); Checking each option one by one, in option (d):

$\frac{a_1}{a_2} = 1,\quad \frac{b_1}{b_2} = -1$

i.e; $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, system of equations given in option (d) has unique solution

10. (b); $2x - ky + 3 = 0$

$y = \frac{2x+3}{k} = \frac{2}{k}x + \frac{3}{k}$

$\text{slope} = \frac{2}{k} = m,\quad 3x + 2y = 1$

$y = -\frac{3}{2}x + \frac{1}{2},$

For the given lines to be parallel, slopes must be equal.

$m_1 = \frac{2}{k} = -\frac{3}{2} = m_2 \quad \Rightarrow \quad k = -\frac{4}{3}$

11. (a); The given equations are:
$5x + 2y - k = 0$ ...(i)
$10x + 4y - 3 = 0$ ...(ii)

$\frac{a_1}{a_2} = \frac{5}{10} = \frac{1}{2},\quad \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2},\quad \frac{c_1}{c_2} = \frac{-k}{-3} = \frac{k}{3}$

For the given system to have infinitely many solutions:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{k}{3} = \frac{1}{2} \quad \Rightarrow \quad k = \frac{3}{2}$

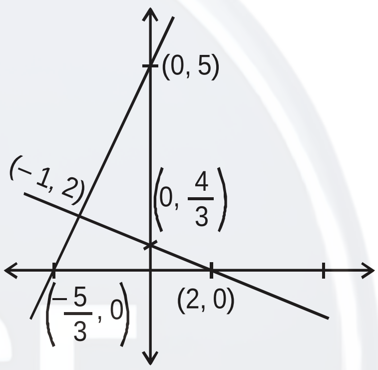
12. (b); Equation (i) $\rightarrow 2x + 3y = 4$

At $x = 0,\ y = \frac{4}{3},\quad$ At $y = 0,\ x = 2$

Equation (ii) $\rightarrow 3x - y = -5$

At $x = 0,\ y = 5,\quad$ At $y = 0,\ x = -\frac{5}{3}$

The graph of the above equations is as follows:



The given lines intersect at (– 1, 2).

So, Required ratio $= \frac{\frac{1}{2}\left(2 - \left(-\frac{5}{3}\right)\right) \times 2}{\frac{1}{2}\left(5 - \frac{4}{3}\right) \times 1} = 2:1$

$= 2:1$

13. (d); abscissa = positive, ordinate = negative so point will lie in fourth quadrant

14. (b); Distance between the given points

$= \sqrt{(b-0)^2 + (0-a)^2} = \sqrt{a^2 + b^2}$

15. (a);

16. (c); Write the equation in the form
$y = mx + c$
$3x + 7y + 8 = 0$

$y = \frac{-3x - 8}{7} = -\frac{3}{7}x - \frac{8}{7}$

$\text{slope} = -\frac{3}{7}$

17. (b);

18. (a); Points A and B both satisfy the equation given in option (a) only.

19. (c); Equation of line formed by the points P(2, 3) and R(6, 7) is:

$$y-3=\left(\frac{7-3}{6-2}\right)(x-2) \Rightarrow y-3=\frac{4}{4}(x-2)$$

$\Rightarrow$ y = x + 1

Now, Q should satisfy the above equation for the three points to be collinear

$\Rightarrow$ a = 5 + 1 = 6

20. (a); Let the ratio be k : 1

x– coordinate of a point lying on y-axis must be 0.

$$\frac{7k-3}{k+1}=0 \Rightarrow k=\frac{3}{7}$$

So, required ratio = 3 : 7

21. (c); x + 3y – 8 = 0

$$\Rightarrow y=\frac{-x+8}{3}=-\frac{1}{3}x+\frac{8}{3}$$

$$\therefore \text{ slope}=-\frac{1}{3}$$

ax + 12y + 5 = 0

$$y=-\frac{ax}{12}-\frac{5}{12}$$

$$\text{Slope}=-\frac{a}{12}$$

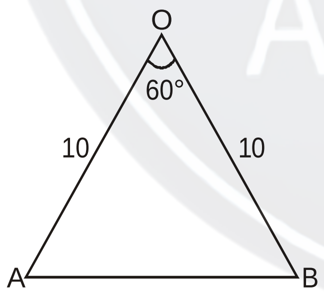
For the lines to be parallel, slopes must be equal

$$\Rightarrow -\frac{1}{3}=-\frac{a}{12} \Rightarrow a=4$$

22. (a); The required coordinates are:

$$\frac{2\times\frac{11}{2}+3\times3}{2+3}, \frac{2\times\frac{21}{2}+3\times(-2)}{2+3}=(4, 3)$$

23. (b);

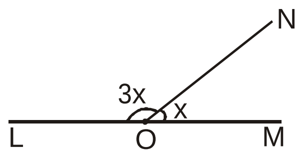


OA and OB are two legs of the compass.

Clearly, the above triangle is an equilateral triangle.

So, distance between end points of the legs is: 10 cm

24. (a);



According to the figure:

4x = 180° $\Rightarrow$ x = 45° = $\angle$MON

25. (b); Let $A=(4, 3)$, $B=(7, -1)$ and $C=(9, 3)$

then $AB=\sqrt{(7-4)^2+(-1-3)^2}=\sqrt{9+16}=5$

$$BC=\sqrt{(9-7)^2+(3+1)^2}=\sqrt{20}=2\sqrt{5}$$

$$CA=\sqrt{(4-9)^2+(3-3)^2}=\sqrt{5^2}=5$$

Therefore it is an isosceles triangle.

26. (b); The required coordinates are

$$\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$$

$(x_1, y_1) = (2, 4)$ $(x_2, y_2) = (6, 8)$

m : n = 5 : 3

$$x=\frac{5\times6-3\times2}{5-3}=\frac{24}{2}=12$$

$$y=\frac{5\times8-3\times4}{5-3}=\frac{28}{2}=14$$

The required coordinates are (12, 14)

27. (a); Coordinate of centroid of ΔABC be (x, y) then

$$(x, y)=\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$=\left(\frac{3+2-2}{3}, \frac{1+3+2}{3}\right)=(1, 2)$$

28. (c); $AB=\sqrt{(1+2)^2+1^2}=\sqrt{10}$

$$BC=\sqrt{(4-1)^2+3^2}=\sqrt{9+9}=\sqrt{18}$$

$$CD=\sqrt{(1-4)^2+(2-3)^2}=\sqrt{9+1}=\sqrt{10}$$

$$DA=\sqrt{(-2-1)^2+(2+1)^2}=\sqrt{9+9}=\sqrt{18}$$

The opposite sides are equal, Hence, it is a parallelogram

29. (a); Slope of line $=\frac{y_2-y_1}{x_2-x_1}=\frac{7-5}{9-7}=1$

30. (a); m_1 = Slope of line AB = $\frac{4+2}{3-1}=\frac{6}{2}=3$

m_2 = Slope of line BC = $\frac{7-4}{4-3}=3$

$m_1 = m_2$

AB is parallel to BC and B is common. Therefore, it is a straight line.

Moderate

1. (a); Slope $= \frac{7-3}{-4-2} = \frac{4}{-6} = \frac{-2}{3}$

2. (c); Area of ΔPQR when the vertices are $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$ is given by the formula;

$$\text{Area} = \frac{1}{2} x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$$

So, area of ΔPQR =

$$\frac{1}{2}[4(8-(-4)) + (-3)(-4-5) + 3(5-8)]$$

$$= \frac{1}{2}|48 + 27 + (-9)|$$

$$= \frac{1}{2} \times 66 = 33 \text{ Sq. units.}$$

3. (d); Co-ordinates of the centroid of ΔPQR are:

$$\left(\frac{-2+9+8}{3}, \frac{0-3+3}{3}\right) \equiv (5, 0)$$

4. (a); Slope of the joining the points $(\sqrt{3}, 1)$ and $(\sqrt{15}, \sqrt{5})$ = $\tan\theta$

$$= \frac{\sqrt{5}-1}{\sqrt{15}-\sqrt{3}} = \frac{(\sqrt{5}-1)}{\sqrt{3}(\sqrt{5}-1)} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ The required angle = 30°

5. (d); According to the question:
$(x-a)^2 + y^2 + (x+a)^2 + y^2 = 2b^2$
$\Rightarrow \quad x^2 + a^2 = b^2 - y^2$

6. (b); Let the co-ordinate of R be (x, y).
According to the question:

$$\frac{-1+5+x}{3} = 4 \quad \Rightarrow \quad x = 8$$

$$\frac{0-2+y}{3} = 0 \quad \Rightarrow \quad y = 2$$

So, Co-ordiantes of R is
(8, 2)

7. (b); Let the ratio be K : 1
on the x-axis, y-coordinate is zero

$$\Rightarrow \quad \frac{4k-5}{k+1} = 0 \quad \Rightarrow \quad K = \frac{5}{4}$$

So, the required ratio is 5 : 4.

8. (d); For the lines to be perpendicular to each other, product of slopes must be equal to –1

Slope of line PQ $= m_1 = \frac{-7-5}{0+2} = -6$

Slope of line AB $= m_2 = \frac{a+2}{8+4} = \frac{a+2}{12}$

$m_1 m_2 = -1$

$$\Rightarrow \quad -6 \times \left(\frac{a+2}{12}\right) = -1 \quad \Rightarrow \quad a = 0$$

9. (a); The given lines can be represented as:

$$y = \frac{\sqrt{12}\,x}{2} + \frac{9}{2}$$

$$\text{and } y = \frac{x}{\sqrt{3}} - \frac{7}{\sqrt{3}}$$

So, the slopes are:

$$m_1 = \frac{\sqrt{12}}{2}, \quad m_2 = \frac{1}{\sqrt{3}}$$

The tangent of the angle between the given lines:

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{1}{\sqrt{3}}$$

$\Rightarrow \quad \theta = 30°$.

10. (d); Given, straight lines:
$2x + 3y = 5$ ... (i)
$3x - y = 13$ ... (ii)
Solving the above two equations, we get the intersection point:
(4, – 1).
From equation (i):

at $y = 0, x = \frac{5}{2}$.

From the equation (ii);

At $y = 0, x = \frac{13}{3}$

So, the three vertices of the triangle are :

$$(4, -1), \left(\frac{5}{2}, 0\right), \left(\frac{13}{3}, 0\right)$$

$$\text{Area} = \frac{1}{2}\left|4(0-0) + \frac{5}{2}(0+1) + \frac{13}{3}(-1-0)\right|$$

$$= \frac{11}{12} \text{ sq. units.}$$

11. (d); Given straight lines:

$4x - y = 4$ ... (i)

$3x + 2y = 14$ ... (ii)

Solving the above two equations we get the intersection point:

(2, 4)

On the y-axis, x-coordiate is zero.

From (i): At $x = 0$, $y = -4$

From (ii): At $x = 0$, $y = 7$

So, the vertices of the triangle are:

(2, 4), (0, – 4) and (0, 7)

$$\text{Area} = \frac{1}{2}\left|2(-4-7)+0+0\right| = 11 \text{ sq. units.}$$

12. (b); Given straight lines:

$2x - 3y = -6$ ... (i)

$2x + 3y = 18$ ... (ii)

$y = 1$ ... (iii)

Form (i) and (iii):.

$2x - 3 = -6$

$$\Rightarrow \quad x = \frac{-3}{2}$$

From (ii) and (iii).

$2x + 3 = 18$

$$\Rightarrow \quad x = \frac{15}{2}$$

From (i) and (ii):

$x = 3$

$y = 4$

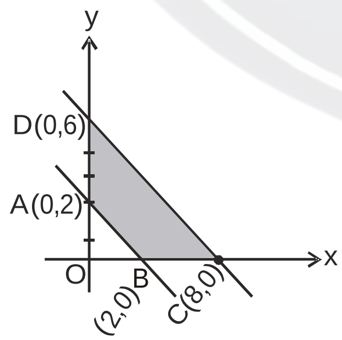
So, the vertices of the triangle are:

$\left(\frac{-3}{2}, 1\right), \left(\frac{15}{2}, 1\right)$ and (3, 4).

$$\text{Area} = \frac{1}{2}\left|\frac{-3}{2}(1-4)+\frac{15}{2}(4-1)+3(1-1)\right|$$

$$= \frac{27}{2} \text{ sq. units.}$$

13. (a);



$$\text{Area of } \Delta DOC = \frac{1}{2} \times 8 \times 6 = 24 \text{ sq. units.}$$

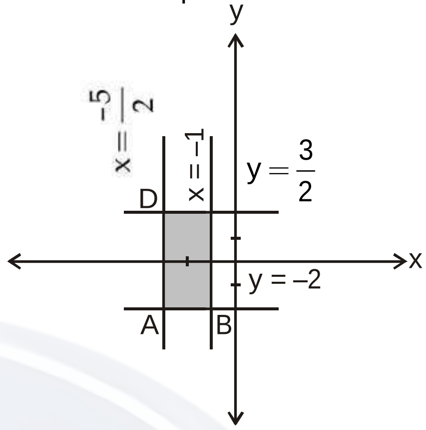
$$\text{Area of } \Delta AOB = \frac{1}{2} \times 2 \times 2 = 2 \text{ sq. units.}$$

Area of Quad. ABCD =

Area of ΔDOC – Area of ΔAOB

= 24 – 2 = 22 sq. units.

14. (b);

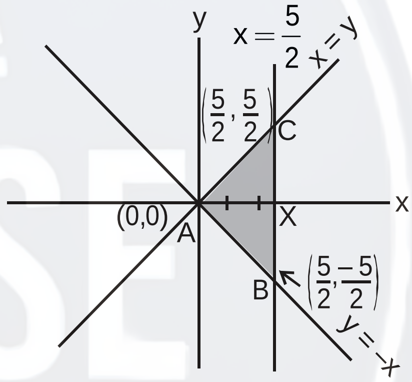


The quadrilateral formed by the given lines is a rectangle.

Area = L × B

$$= \frac{7}{2} \times \frac{3}{2} = \frac{21}{4} \text{ sq. units}$$

15. (c);



$$\text{Length of base BC of } \Delta ABC = \frac{5}{2}+\frac{5}{2} = 5 \text{ units}$$

$$\text{Height} = AX = \frac{5}{2} \text{ units.}$$

$$\text{Area} = \frac{1}{2} \times 5 \times \frac{5}{2} = \frac{25}{4} \text{ sq. units.}$$

16. (a); Given equations:

$2x + 3y - 7 = 0$ ... (i)

$2ax + (a + b)y - 28 = 0$ ... (ii)

For infinitely many solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

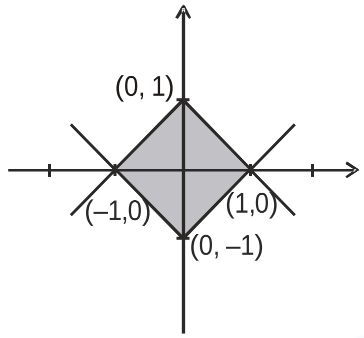
$$\Rightarrow \quad \frac{2}{2a} = \frac{3}{a+b} = \frac{-7}{-28}$$

$$\Rightarrow \quad \frac{1}{a} = \frac{7}{28} = \frac{1}{4} \Rightarrow a = 4$$

$$\text{and, } \frac{1}{a} = \frac{3}{a+b} \Rightarrow b = 2a = 8$$

17. (a); Given equations: $y = |x| - 1$

$y = 1 - |x|$



Each side of the figure formed by the given equations = $\sqrt{2}$ units

Area = $\sqrt{2} \times \sqrt{2} = 2$ sq. units.

18. (b); Fig

Given $AC^2 = AB \times CB$

$\Rightarrow \quad x^2 = 2 \times (2 - x)$

$x^2 = 4 - 2x$

$x^2 + 2x - 4 = 0$

$ax^2 + bx + c = 0$

$$x = \frac{-2 \pm \sqrt{4 + 16}}{2 \times 1} = 1 \pm \sqrt{5}$$

Now, BC = $2 - (-1 \pm \sqrt{5}) = 3 - \sqrt{5}$

$\left(3 + \sqrt{5} > AB, \text{ which is not possible}\right)$

19. (c); Sum of all angle exculded angle of triangle are =

$360° + 360° + 360° - 180°$

$= 1080° - 180° = 900°$

20. (a); Slope of AB = $\frac{5+3}{2-x} = \frac{8}{2-x} = \tan 135°$

$\tan(180° - 45°) = \frac{8}{2-x}$

$-\tan 45° = \frac{8}{2-x} \quad \Rightarrow \quad -1 = \frac{8}{2-x}$

$\Rightarrow \quad 2 - x = -8$

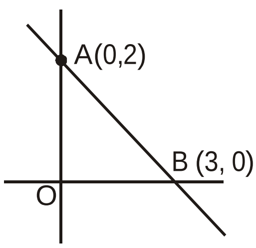
$x = 8 + 2$

$x = 10$

21. (b); $2x + 3y = 6$

$\frac{2x}{6} + \frac{3y}{6} = 1$

$\frac{x}{3} + \frac{y}{2} = 1$



Therefore It intercept at x-axis at 3 and intercept at y-axis = 2, Area of $\Delta OAB = \frac{1}{2} \times 3 \times 2 = 3$ sq. units

22. (a); $x + 2y = 9$

$kx + 4y = -5$

For equation of line to be parallel $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\frac{1}{k} = \frac{2}{4} \quad k = 2$

23. (d); Slope of the line made by joining the points A (1, 1) and B(3, 5) = $\frac{5-1}{3-1} = \frac{4}{2} = 2$ and mid point of AB = $\left(\frac{1+3}{2}, \frac{1+5}{2}\right)$ = (2, 3)

Now, slope of the line perpendicular to the line AB = $\frac{-1}{2}$

$\therefore$ Required equation of perpendicular bisector

$(y - 3) = -\frac{1}{2}(x - 2)$

$x + 2y - 8 = 0$

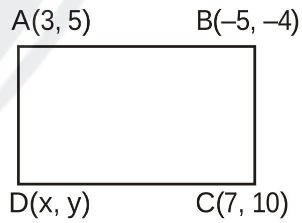
24. (b); Let the fourth vertex be (x, y)

Diagonal of parallelogram intersect at mid point

Therefore

midpoint of AC = Mid-point of BD

$$\left(\frac{3+7}{2}, \frac{5+10}{2}\right) = \left(\frac{-5+x}{2}, \frac{-4+y}{2}\right)$$



$\Rightarrow \quad 5 = \frac{-5+x}{2}$

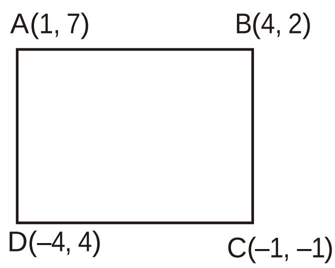
$x = 10 + 5 = 15$

$\frac{5+10}{2} = \frac{-4+y}{2}$

$y = 19$

Fourth vertex will be (15, 19)

25. (b);



$AB = \sqrt{(4-1)^2 + (2-7)^2} = \sqrt{9+25} = \sqrt{34}$

$BC = \sqrt{(-1-4)^2 + (-1-2)^2} = \sqrt{5^2 + 3^2} = \sqrt{34}$

$CD = \sqrt{(-4+1)^2 + (+4+1)^2} = \sqrt{3^2 + 5^2} = \sqrt{34}$

$DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{5^2 + 3^2} = \sqrt{34}$

$AC = \sqrt{(-1-1)^2 + (-1-7)^2} = \sqrt{2^2 + 8^2} = \sqrt{68}$

$BD = \sqrt{(-4-4)^2 + (4-2)^2} = \sqrt{8^2 + 2^2} = \sqrt{68}$

Given, AB = BC = CD = DA and AC = BD

Therefore it is a square.

26. (b); Since, P, Q and R are collinear

Area of $\Delta PQR = 0$

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

Given $x_1 = a$, $x_2 = 0$

$x_3 = 1$, $y_1 = 0$, $y_2 = b$ and $y_3 = 1$

$$\frac{1}{2}[a(b-1) + 0(1-0) + 1(0-b)] = 0$$

$ab - a - b = 0$

$ab = a + b$

$$1 = \frac{1}{a} + \frac{1}{b}$$

Coordinate Geometry Mid-Point Formula

→ **Midpoint Formula:** This formula determines the midpoint of a line joining two points (x1, y1) and (x2, y2). The midpoint's coordinates are the averages of the x and y coordinates of the given points.

Coordinate Geometry Section Formula

→ **Section Formula:** The section formula identifies the coordinates of a point that divides the line segment between two points (x1, y1) and (x2, y2) in a given ratio m:n.

Coordinate Geometry Centroid Formula

→ **Centroid Formula:** The centroid of a triangle formed by vertices A (x1, y1), B (x2, y2), and C (x3, y3) can be found using the formula: Centroid (x, y) = ((x1 + x2 + x3) / 3, (y1 + y2 + y3) / 3)

Coordinate Geometry Area of a Triangle Formula

→ **Area of a Triangle Formula:** The area of a triangle with vertices A (x1, y1), B (x2, y2), and C (x3, y3) is computed using the formula: Area = 0.5 * |x1(y2 - y3) + x2(y3 - y1) + x3(y1 - y2)|