Chapter - 3

Surds and Indices

Surds and Indices Formulas

(a + b)(a – b) = (a2 – b2)
$(a + b)^2 = (a^2 + b^2 + 2ab)$
$(a - b)^2 = (a^2 + b^2 - 2ab)$
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
$(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
$(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
$(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
When a + b + c = 0, then $a^3 + b^3 + c^3 = 3abc$.

Surds and Indices Rules

Rule Name	Surds Rule	Indices Rule
Multiplication Rule	$a^n * b^n = (a*b)^n$	$a^n * a^m = a^{(m+n)}$
Division Rule	$a^n / b^n = (a/b)^n$	$a^m / a^n = a^{(m-n)}$
Power Rule	$(a^n)^m = (a)^{nm}$ $n\sqrt{a} = a^{(1/n)}$	$a^{(n^m)}) = a^{n^m}$ $a^{-n} = 1/(a^n)$

Foundation

Solutions

1. (c); $3^{x-1}(3 - 1) = 18$
$3^{x-1} = 9 = 3^2$
$x - 1 = 2 \Rightarrow x = 3$ so, $x^x = 27$

2. (b); $a^{2x+2} = a^0$
$2x = -2 \Rightarrow x = -1$

3. (d); $\frac{12^{-4}}{12^{-4}} = 1$

4. (a); $7^{?} = \frac{7^{8.9} \times (7^2)^{4.8}}{(7^3)^{1.7}} = \frac{7^{18.5}}{7^{5.1}} = 7^{13.4}$

? = 13.4

5. (c); $2^{4 \times \frac{1}{2} \times ?} = 2^8$

? = 4

6. (b); $(16)^{?} = \frac{(16)^9 \times (16)^3}{(16)^4} = 16^{9+3-4} = 16^8$

? = 8

7. (d); $\frac{(42 \times 229)}{(9261)^{\frac{1}{3}}} = \frac{42 \times 229}{21} = 458$

8. (d); $\left(\frac{32}{100000}\right)^{\frac{2}{5}} = \left\{\left(\frac{2}{10}\right)^5\right\}^{\frac{2}{5}} = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$

9. (d); $\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{(\sqrt{7}-\sqrt{5})(\sqrt{7}-\sqrt{5})}{2} = 6-\sqrt{35}$

on comparision, $a = 6$ and $b = -1$

$a - b = 6 - (-1) = 7$

10. (d); $\sqrt[3]{P(P^2+3P+3)+1} = \sqrt[3]{P^3+3P^2+3P+1}$

$= \sqrt[3]{(P+1)^3} = P+1$

P = 124, = P + 1 = 125

11. (d); $\left(\frac{p}{q}\right)^{n-1} = \left(\frac{p}{q}\right)^{3-n}$

$n - 1 = 3 - n, \quad 2n = 4 \Rightarrow n = 2$

12. (c); ? = 8 ÷ 2 + 24 = 28

13. (d); $3 + \frac{1}{\sqrt{3}} + \frac{(3-\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})} + \frac{(\sqrt{3}+3)}{(\sqrt{3}-3)(\sqrt{3}+3)} - 3$

$= 3 + \frac{1}{\sqrt{3}} + \frac{3-\sqrt{3}-\sqrt{3}-3}{6} - 3 = 3 + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} - 3 = 0$

14. (c); $(16)^{\frac{5}{4}} = (2^4)^{\frac{5}{4}} = 32$

15. (a); $\left(\frac{243}{32}\right)^{\frac{3}{5}} = \left(\frac{3^5}{2^5}\right)^{\frac{3}{5}} = \frac{27}{8}$

16. (c); $(243)^{0.2} = (3^5)^{0.2} = 3$

17. (c); $17^{3.5+7.3-4.2} = 17^x$

x = 6.6

18. (c); $17^2 = 17^{\frac{x}{5}} \Rightarrow \frac{x}{5} = 2 \Rightarrow x = 10$

19. (a); $\left[\left\{\frac{1}{4}\right\}^{-2}\right]^{-1} = \left\{\frac{1}{\frac{1}{16}}\right\}^{-1} = 16^{-1} = \frac{1}{16}$

20. (a); $10^{200-196} = 10000$

21. (c); $\left(\frac{1}{5}\right)^{3a} = 0.008 = \frac{8}{1000} = \left(\frac{1}{5}\right)^3$

$3a = 3 \Rightarrow a = 1$

$(0.25)^a = 0.25$

22. (b); $\sqrt{(\sqrt{3})^2+(\sqrt{2})^2+2\sqrt{3}\sqrt{2}} - \frac{1}{\sqrt{(\sqrt{3})^2+(\sqrt{2})^2-2\sqrt{3}\sqrt{2}}}$

$= \sqrt{3}+\sqrt{2} - \frac{1}{\sqrt{3}-\sqrt{2}} = \sqrt{3}+\sqrt{2}-\sqrt{3}-\sqrt{2} = 0$

23. (b); $\frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}} + \frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}}$

$= \frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^a+x^b+x^c} + \frac{x^c}{x^a+x^b+x^c} = 1$

24. (c); $P^{b^2-c^2} \cdot P^{c^2-a^2} \cdot P^{a^2-b^2} = P^{b^2-c^2+c^2-a^2+a^2-b^2} = 1$

25. (a); $\sqrt{5+\sqrt[3]{x}} = 3$

$\sqrt[3]{x} = 9-5 \Rightarrow x = 4^3 = 64$

26. (c); $\left[\left\{x^{-\frac{3}{5}\times\frac{1}{5}}\right\}^{-\frac{5}{3}}\right]^5 = x^{\frac{-3}{5}\times\frac{1}{5}\times\frac{-5}{3}\times 5} = x$

27. (c); 56 = 8 × 7

so, $\sqrt{56+\sqrt{56+\sqrt{56+.....}}} = 8$

$= 8 \div 2^2 = 2$

28. (a); $b = a^x$

$b^y = a^{xy}$

$b^{yz} = a^{xyz}$

$c^z = a$

29. (d); $2^4 \times 2^{3(n+2)} = 2^m$

$2^m = 2^{(3n+10)}$

3n + 10 = m

30. (a); $x^2 + y^2 = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2 + \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)^2$

$= \frac{4+2\sqrt{3}}{4-2\sqrt{3}} + \frac{4-2\sqrt{3}}{4+2\sqrt{3}}$

$= \frac{(4+2\sqrt{3})^2+(4-2\sqrt{3})^2}{(4-2\sqrt{3})(4+2\sqrt{3})}$

$= \frac{16+12+16\sqrt{3}+16+12-16\sqrt{3}}{16-12} = \frac{56}{4} = 14$

Moderate

1. (c); Expression = $\dfrac{16\times2^{n+1}-4\times2^{n}}{16\times2^{n+2}-2\times2^{n+2}}$

$=\dfrac{2^{4}\times2^{n+1}-2^{2}\times2^{n}}{2^{4}\times2^{n+2}-2\times2^{n+2}}=\dfrac{2^{n+5}-2^{n+2}}{2^{n+6}-2^{n+3}}$

$=\dfrac{2^{n+5}-2^{n+2}}{2\times2^{n+5}-2\times2^{n+2}}=\dfrac{2^{n+5}-2^{n+2}}{2(2^{n+5}-2^{n+2})}=\dfrac{1}{2}$

2. (c); Expression = $\dfrac{2^{2^3}\div(2^2)^3\times2^{-2}}{4^{2^3}\div(4^2)^3\times4^{-2}}$

$=\dfrac{2^{2\times2\times2}\div2^{2\times3}\times2^{-2}}{4^{2\times2\times2}\div4^{2\times3}\times4^{-2}}$

$=\dfrac{2^{8}\div2^{6}\times2^{-2}}{4^{8}\div4^{6}\times4^{-2}}=\dfrac{2^{8}\times\frac{1}{2^{6}}\times\frac{1}{2^{2}}}{4^{8}\times\frac{1}{4^{6}}\times\frac{1}{4^{2}}}=1$

3. (b); Expression

$=\dfrac{(0.6)^{0}-(0.1)^{-1}}{\left(\frac{3}{2^{3}}\right)^{-1}\left(\frac{3}{2}\right)^{3}+\left(-\frac{1}{3}\right)^{-1}}+\left(\frac{8}{27}\right)^{-\frac{1}{3}}$

$=\dfrac{1-\left(\frac{1}{10}\right)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^{3}+\left(-\frac{1}{3}\right)^{-1}}+\left(\frac{8}{27}\right)^{-\frac{1}{3}}$

$=\dfrac{1-10}{\frac{8}{3}\times\frac{27}{8}+(-3)}+\left(\frac{27}{8}\right)^{\frac{1}{3}}$

$=\dfrac{-9}{9-3}+\left[\left(\frac{3}{2}\right)^{3}\right]^{\frac{1}{3}}=-\dfrac{9}{6}+\dfrac{3}{2}=-\dfrac{3}{2}+\dfrac{3}{2}=0$

4. (d); $\dfrac{(81)^{4x}\times(27)^{x}\times9^{7}}{(729)^{x+2}}=3^{9}$

$\Rightarrow\dfrac{(3^{4})^{4x}\times(3^{3})^{x}\times(3^{2})^{7}}{(3^{6})^{x+2}}=3^{9}$

$\Rightarrow\dfrac{3^{16x}\times3^{3x}\times3^{14}}{3^{6x+12}}=3^{9}$

$\Rightarrow\dfrac{3^{16x+3x+14}}{3^{6x+12}}=3^{9}\quad\Rightarrow\quad3^{19x+14-6x-12}=3^{9}$

$3^{13x+2}=3^{9}\quad\Rightarrow\quad13x+2=9$

$13x=9-2=7\quad\Rightarrow\quad x=\dfrac{7}{13}$

5. (c); The equations are
$2^a + 3^b = 17$ and $2^a \times 2^2 - 3^b \times 3 = 5$
Let $x = 2^a$ and $y = 3^b$
then, $x + y = 17$ ... (i)
and $4x - 3y = 5$ ... (ii)
By equation (i) × 3 + (ii),

$$\begin{array}{rcrcr} 3x & + & 3y & = & 51\\ 4x & - & 3y & = & 5\\ \hline 7x & & & = & 56 \end{array}$$

From equation (i),
$y = 17 - x = 17 - 8 = 9$
$\therefore x = 8$ and $y = 9$
$\therefore x = 2^a = 8 \Rightarrow 2^a = 2^3 \Rightarrow a = 3$
and $y = 3^b = 9 \Rightarrow 3^b = 3^2 \Rightarrow b = 2$
$\therefore a = 3$ and $b = 2$

6. (a); Expression = $\dfrac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}-\dfrac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}+\dfrac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$

$=\dfrac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}\times\dfrac{\sqrt{6}-\sqrt{3}}{\sqrt{6}-\sqrt{3}}-\dfrac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}\times\dfrac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$

$+\dfrac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}\times\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

[Rationalising the respective denominators]

$=\dfrac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{6-3}-\dfrac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{6-2}+\dfrac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{3-2}$

$=\sqrt{2}(\sqrt{6}-\sqrt{3})-\sqrt{3}(\sqrt{6}-\sqrt{2})+\sqrt{6}(\sqrt{3}-\sqrt{2})$

$=\sqrt{12}-\sqrt{6}-\sqrt{18}+\sqrt{6}+\sqrt{18}-\sqrt{12}=0$

7. (d); Let $x=\dfrac{2(\sqrt{2}+\sqrt{6})}{3\sqrt{2+\sqrt{3}}}$

Squaring both sides, $x^{2}=\left[\dfrac{2(\sqrt{2}+\sqrt{6})}{3\sqrt{2+\sqrt{3}}}\right]^{2}$

$x^{2}=\dfrac{4(\sqrt{2}+\sqrt{6})^{2}}{(3\sqrt{2+\sqrt{3}})^{2}}\quad\Rightarrow\quad x^{2}=\dfrac{4(\sqrt{2}+\sqrt{6})^{2}}{9(2+\sqrt{3})}$

$x^{2}=\dfrac{4(2+6+2\times\sqrt{2}\times\sqrt{6})}{9(2+\sqrt{3})}$

$x^2 = \frac{4(8+2\sqrt{12})}{9(2+\sqrt{3})} \Rightarrow x^2 = \frac{4(8+2\sqrt{2^2\times 3})}{9(2+\sqrt{3})}$

$x^2 = \frac{4(8+4\sqrt{3})}{9(2+\sqrt{3})} \Rightarrow x^2 = \frac{16(2+\sqrt{3})}{9(2+\sqrt{3})} = \frac{16}{9}$

$x = \frac{4}{3}$

8. (c); $x = \frac{\sqrt{5}-2}{\sqrt{5}+2}$

$= \frac{(\sqrt{5}-2)^2}{(\sqrt{5}+2)(\sqrt{5}-2)} = \frac{5+4-4\sqrt{5}}{5-4} = 9-4\sqrt{5}$

$\therefore \frac{1}{x} = 9+4\sqrt{5}, \quad x^4 + x^{-4} = x^4 + \frac{1}{x^4}$

$= \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = \left[\left(x+\frac{1}{x}\right)^2 - 2\right]^2 - 2$

$= \left[(9+4\sqrt{5}+9-4\sqrt{5})^2 - 2\right]^2 - 2$

$= [(18)^2 - 2]^2 - 2 = (322)^2 - 2 = 103682$ (integer)

9. (b); Expression $= \sqrt{\frac{19+8\sqrt{3}}{7-4\sqrt{3}}}$

$= \sqrt{\frac{19+2\times 4\times\sqrt{3}}{7-2\times 2\times\sqrt{3}}} = \sqrt{\frac{16+3+2\times 4\times\sqrt{3}}{4+3-2\times 2\times\sqrt{3}}}$

$= \sqrt{\frac{(4)^2+(\sqrt{3})^2+2\times 4\times\sqrt{3}}{(2)^2+(\sqrt{3})^2-2\times 2\times\sqrt{3}}} = \sqrt{\frac{(4+\sqrt{3})^2}{(2-\sqrt{3})^2}}$

$\left[\because (a\pm b)^2 = a^2+b^2\pm 2ab\right]$

$= \frac{4+\sqrt{3}}{2-\sqrt{3}}$

Rationalising the denominator.

$= \frac{(4+\sqrt{3})}{(2-\sqrt{3})} \times \frac{(2+\sqrt{3})}{(2+\sqrt{3})} = \frac{8+2\sqrt{3}+4\sqrt{3}+3}{4-3}$

$= 11+6\sqrt{3}$

10. (c); First term $= (28+10\sqrt{3})^{\frac{1}{2}}$

$= (28+2\times 5\times\sqrt{3})^{\frac{1}{2}}$

$= (25+3+2\times 5\times\sqrt{3})^{\frac{1}{2}}$

$= \left[(5)^2+(\sqrt{3})^2+2\times 5\times\sqrt{3}\right]^{\frac{1}{2}}$

$= \left[(5+\sqrt{3})^2\right]^{\frac{1}{2}} = 5+\sqrt{3}$

Second term $= (7-4\sqrt{3})^{-\frac{1}{2}} = (7-2\times 2\times\sqrt{3})^{-\frac{1}{2}}$

$= (4+3-2\times 2\times\sqrt{3})^{-\frac{1}{2}}$

$= \left[(2)^2+(\sqrt{3})^2-2\times 2\times\sqrt{3}\right]^{-\frac{1}{2}}$

$= \left[(2-\sqrt{3})^2\right]^{-\frac{1}{2}} = (2-\sqrt{3})^{2\times\frac{-1}{2}}$

$= (2-\sqrt{3})^{-1} = \frac{1}{(2-\sqrt{3})}$

$= \frac{1\times(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{(2+\sqrt{3})}{4-3} = 2+\sqrt{3}$

$\therefore$ Expression

$= (5+\sqrt{3})-(2+\sqrt{3}) = 5+\sqrt{3}-2-\sqrt{3} = 3$

11. (c); Expression $= \sqrt{6-4\sqrt{3}+\sqrt{16-8\sqrt{3}}}$

$= \sqrt{6-4\sqrt{3}+\sqrt{16-2\times 2\sqrt{3}\times 2}}$

$= \sqrt{6-4\sqrt{3}+\sqrt{12+4-2\times 2\sqrt{3}\times 2}}$

$= \sqrt{6-4\sqrt{3}+\sqrt{(2\sqrt{3})^2+(2)^2-2\times 2\sqrt{3}\times 2}}$

$= \sqrt{6-4\sqrt{3}+\sqrt{(2\sqrt{3}-2)^2}}$

$= \sqrt{6-4\sqrt{3}+2\sqrt{3}-2}$

$= \sqrt{4-2\sqrt{3}} = \sqrt{3+1-2\times\sqrt{3}\times 1}$

$= \sqrt{(\sqrt{3})^2+(1)^2-2\times\sqrt{3}\times 1}$

$= \sqrt{(\sqrt{3}-1)^2} = \sqrt{3}-1$

12. (b); $x = \sqrt{\frac{5+2\sqrt{6}}{5-2\sqrt{6}}}$ Rationalising,

$x = \sqrt{\frac{5+2\sqrt{6}}{5-2\sqrt{6}} \times \frac{5+2\sqrt{6}}{5+2\sqrt{6}}}$

$= \sqrt{\frac{(5+2\sqrt{6})^2}{25-24}} = 5+2\sqrt{6}$

$\therefore x^2(x-10)^2$

$=(5+2\sqrt{6})^2(5+2\sqrt{6}-10)^2$

$=(5+2\sqrt{6})^2(2\sqrt{6}-5)^2$

$=(25+24+20\sqrt{6})(24+25-20\sqrt{6})$

$=(49+20\sqrt{6})(49-20\sqrt{6})$

$=(49)^2-(20\sqrt{6})^2=2401-2400=1$

13. (a); $\dfrac{\sqrt{4-\sqrt{7}}}{\sqrt{8+3\sqrt{7}}-2\sqrt{2}}=\dfrac{\sqrt{8-2\sqrt{7}}}{\sqrt{16+6\sqrt{7}}-4}$

(Multiplying numerator and denominator by $\sqrt{2}$)

$=\dfrac{\sqrt{(1)^2+(\sqrt{7})^2-2\times1\times\sqrt{7}}}{\sqrt{(3)^2+(\sqrt{7})^2+2.3.\sqrt{7}}-4}$

$\{\therefore 8=1+7=1^2+(\sqrt{7})^2$ and $16=9+7=3^2+(\sqrt{7})^2\}$

$=\dfrac{\sqrt{(\sqrt{7}-1)^2}}{\sqrt{(\sqrt{7}+3)^2}-4}$ $\{\because a^2+b^2-2ab=(a-b)^2\}$

$=\dfrac{\sqrt{7}-1}{\sqrt{7}+3-4}=\dfrac{\sqrt{7}-1}{\sqrt{7}-1}=1$

14. (d); Expression $=\dfrac{(625)^{6.25}\times(25)^{2.6}}{(625)^{6.75}\times(5)^{1.2}}$

$=\dfrac{(5^4)^{6.25}\times(5^2)^{2.6}}{(5^4)^{6.75}\times(5)^{1.2}}=\dfrac{(5)^{4\times6.25}\times(5)^{2\times2.6}}{(5)^{4\times6.75}\times(5)^{1.2}}$

$=\dfrac{(5)^{25}\times(5)^{5.2}}{(5)^{27}\times(5)^{1.2}}=\dfrac{(5)^{25+5.2}}{(5)^{27+1.2}}=\dfrac{(5)^{30.2}}{(5)^{28.2}}$

$=(5)^{30.2-28.2}=(5)^2=25$

15. (a); Expression $=\dfrac{2+\sqrt{3}}{2-\sqrt{3}}$

$=\dfrac{(2+\sqrt{3})}{(2-\sqrt{3})}\times\dfrac{(2+\sqrt{3})}{(2+\sqrt{3})}$

[Multiplying N^r and D^r by $(2+\sqrt{3})$]

$=\dfrac{(2+\sqrt{3})^2}{(2)^2-(\sqrt{3})^2}$ $[\because (a+b)(a-b)=a^2-b^2]$

$=\dfrac{4+3+2\times2\sqrt{3}}{4-3}=7+4\sqrt{3}$

16. (b); Expression $=\dfrac{(0.3)^{\frac{1}{3}}\left(\frac{1}{27}\right)^{\frac{1}{4}}(9)^{\frac{1}{6}}(0.81)^{\frac{2}{3}}}{(0.9)^{\frac{2}{3}}(3)^{-\frac{1}{2}}(243)^{-\frac{1}{4}}}$

$=\dfrac{(0.3)^{\frac{1}{3}}(9)^{\frac{1}{6}}(0.81)^{\frac{2}{3}}\times(3)^{\frac{1}{2}}(243)^{\frac{1}{4}}}{(0.9)^{\frac{2}{3}}\times(27)^{\frac{1}{4}}}$

$=\dfrac{(0.3)^{\frac{1}{3}}(3^2)^{\frac{1}{6}}(0.9)^{2\times\frac{2}{3}}\times(3)^{\frac{1}{2}}(3^5)^{\frac{1}{4}}}{(0.9)^{\frac{2}{3}}\times(3^3)^{\frac{1}{4}}}$

$=\dfrac{(0.3)^{\frac{1}{3}}3^{\frac{1}{3}}\times(0.9)^{\frac{4}{3}}\times3^{\frac{1}{2}}\times3^{\frac{5}{4}}}{(0.9)^{\frac{2}{3}}\times3^{\frac{3}{4}}}$

$=(0.3)^{\frac{1}{3}}\times(0.9)^{\frac{4}{3}-\frac{2}{3}}\times3^{\frac{1}{3}+\frac{1}{2}+\frac{5}{4}-\frac{3}{4}}$

$=(0.3)^{\frac{1}{3}}\times(0.9)^{\frac{2}{3}}\times3^{\frac{4+6+15-9}{12}}$

$=(0.3)^{\frac{1}{3}}\times(0.3\times3)^{\frac{2}{3}}\times3^{\frac{16}{12}}$

$=(0.3)^{\frac{1}{3}}\times(0.3)^{\frac{2}{3}}\times(3)^{\frac{2}{3}}\times(3)^{\frac{4}{3}}$

$=(0.3)^{\frac{1}{3}+\frac{2}{3}}\times3^{\frac{2}{3}+\frac{4}{3}}=(0.3)^{\frac{1+2}{3}}\times3^{\frac{2+4}{3}}=0.3\times3^2$

$=0.3\times9=2.7$

17. (a); Expression $=\dfrac{2^n\times2^4-2\times2^n}{2\times2^n\times2^3}+\dfrac{1}{2^3}$

$=\dfrac{2^n(2^4-2)}{2^n(2\times2^3)}+\dfrac{1}{2^3}=\dfrac{16-2}{16}+\dfrac{1}{8}=\dfrac{7}{8}+\dfrac{1}{8}=1$

18. (b); $10^{0.48}=x$, $10^{0.7}=y$ and $x^z=y^2$

$\therefore (10^{0.48})^z=(10^{0.7})^2$

$\Rightarrow 10^{0.48z}=10^{1.4} \Rightarrow 0.48z=1.4$

$\Rightarrow z=\dfrac{1.4}{0.48}=\dfrac{140}{48}=\dfrac{35}{12}=2\dfrac{11}{12}$

19. (a); Expression $=(27)^{-\frac{2}{3}}+\left[\left(2^{\frac{-2}{3}}\right)^{-\frac{5}{3}}\right]^{-\frac{9}{10}}$

$=(3^3)^{\frac{-2}{3}}+\left(2^{-\frac{2}{3}\times\frac{-5}{3}}\right)^{-\frac{9}{10}}=3^{3\times-\frac{2}{3}}+2^{\frac{-2}{3}\times\frac{-5}{3}\times\frac{-9}{10}}$

$=3^{-2}+2^{-1}=\dfrac{1}{3^2}+\dfrac{1}{2}=\dfrac{1}{9}+\dfrac{1}{2}=\dfrac{2+9}{18}=\dfrac{11}{18}$

20. (b); First term $= \sqrt{8+3\sqrt{7}} = \sqrt{\frac{(8+3\sqrt{7})\times 2}{2}}$

$= \sqrt{\frac{16+6\sqrt{7}}{2}} = \sqrt{\frac{3^2+(\sqrt{7})^2+2\times 3\times\sqrt{7}}{2}}$

$= \sqrt{\frac{(3+\sqrt{7})^2}{2}} = \frac{3+\sqrt{7}}{\sqrt{2}}$

Second term $= \sqrt{7+3\sqrt{5}} = \sqrt{\frac{(7+3\sqrt{5})\times 2}{2}}$

$= \sqrt{\frac{14+6\sqrt{5}}{2}} = \sqrt{\frac{3^2+(\sqrt{5})^2+2\times 3\times\sqrt{5}}{2}}$

$= \sqrt{\frac{(3+\sqrt{5})^2}{2}} = \frac{3+\sqrt{5}}{\sqrt{2}}$

$\therefore$ Expression $= \frac{3+\sqrt{7}}{\sqrt{2}} - \frac{3+\sqrt{5}}{\sqrt{2}}$

$= \frac{3+\sqrt{7}-3-\sqrt{5}}{\sqrt{2}} = \frac{\sqrt{7}-\sqrt{5}}{\sqrt{2}} = \frac{\sqrt{14}-\sqrt{10}}{2}$

21. (b); First term $= \sqrt{38+5\sqrt{3}} = \sqrt{\frac{(38+5\sqrt{3})\times 2}{2}}$

$= \sqrt{\frac{76+10\sqrt{3}}{2}} = \sqrt{\frac{75+1+2\times 5\sqrt{3}\times 1}{2}}$

$= \sqrt{\frac{(5\sqrt{3}+1)^2}{2}} = \frac{5\sqrt{3}+1}{\sqrt{2}}$

Second term $= \sqrt{3-\sqrt{5}} = \sqrt{(3-\sqrt{5})\times\frac{2}{2}}$

$= \sqrt{\frac{6-2\sqrt{5}}{2}} = \sqrt{\frac{(\sqrt{5})^2+1-2\sqrt{5}}{2}}$

$= \sqrt{\frac{(\sqrt{5}-1)^2}{2}} = \frac{\sqrt{5}-1}{\sqrt{2}}$

$\therefore$ Expression $= \frac{5\sqrt{3}+1}{\sqrt{2}} + \frac{\sqrt{5}-1}{\sqrt{2}}$

$= \frac{5\sqrt{3}+1+\sqrt{5}-1}{\sqrt{2}} = \frac{5\sqrt{3}+\sqrt{5}}{\sqrt{2}}$

$= \frac{5\sqrt{3}+\sqrt{5}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{6}+\sqrt{10}}{2}$

22. (a); First term $= \sqrt{4-\sqrt{7}} = \sqrt{\frac{(4-\sqrt{7})\times 2}{2}}$

$= \sqrt{\frac{8-2\sqrt{7}}{2}} = \sqrt{\frac{(\sqrt{7})^2+1-2\sqrt{7}}{2}}$

$= \sqrt{\frac{(\sqrt{7}-1)^2}{2}} = \frac{\sqrt{7}-1}{\sqrt{2}}$

Second term $= \sqrt{8+3\sqrt{7}} = \sqrt{\frac{(8+3\sqrt{7})\times 2}{2}}$

$= \sqrt{\frac{16+6\sqrt{7}}{2}} = \sqrt{\frac{3^2+(\sqrt{7})^2+2\times 3\times\sqrt{7}}{2}}$

$= \sqrt{\frac{(3+\sqrt{7})^2}{2}} = \frac{3+\sqrt{7}}{\sqrt{2}}$

$\therefore$ Expression $= \frac{\sqrt{7}-1}{\sqrt{2}} + \frac{3+\sqrt{7}}{\sqrt{2}} = \frac{2\sqrt{7}+2}{\sqrt{2}}$

$= \frac{2\sqrt{14}+2\sqrt{2}}{2} = \sqrt{14}+\sqrt{2} = \sqrt{2}(\sqrt{7}+1)$

23. (b); Denominator of expression

$= \sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}$

$= \sqrt{10}+\sqrt{2^2\times 5}+\sqrt{2^2\times 10}-\sqrt{5}-\sqrt{2^4\times 5}$

$= \sqrt{10}+2\sqrt{5}+2\sqrt{10}-\sqrt{5}-4\sqrt{5}$

$= (1+2)\sqrt{10}+(2-1-4)\sqrt{5}$

$= 3\sqrt{10}-3\sqrt{5} = 3(\sqrt{10}-\sqrt{5})$

$\therefore$ Expression $= \frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$

$= \frac{15}{3(\sqrt{10}-\sqrt{5})} = \frac{5}{\sqrt{10}-\sqrt{5}}\times\frac{\sqrt{10}+\sqrt{5}}{\sqrt{10}+\sqrt{5}}$

$= \frac{5(\sqrt{10}+\sqrt{5})}{10-5} = \sqrt{10}+\sqrt{5}$

= 3.162 + 2.236 = 5.398

24. (b); Expression $= \frac{\left(2^{2n}-3.\frac{2^{2n}}{2^2}\right)\left(3^n-\frac{2.3^n}{3^2}\right)}{\frac{3^n}{3^4}(4^n.4^3-2^{2n})}$

$= \frac{3^n.2^{2n}\left(1-\frac{3}{2^2}\right)\left(1-\frac{2}{3^2}\right)}{3^n.2^{2n}\left(\frac{2^6-1}{3^4}\right)}$

$$=\frac{\left(1-\frac{3}{4}\right)\left(1-\frac{2}{9}\right)}{\frac{64-1}{81}}=\frac{\frac{1}{4}\times\frac{7}{9}}{\frac{63}{81}}=\frac{7}{36}\times\frac{81}{63}=\frac{1}{4}.$$

25. (b); Rationalising the denominator,

$$\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}\times\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$$

$$=\frac{\sqrt{2}\times3\sqrt{2}+\sqrt{2}\times2\sqrt{3}+\sqrt{3}\times3\sqrt{2}+\sqrt{3}\times2\sqrt{3}}{(3\sqrt{2})^2-(2\sqrt{3})^2}$$

$$=\frac{3\sqrt{2\times2}+2\sqrt{2\times3}+3\sqrt{2\times3}+2\sqrt{3\times3}}{9\times2-4\times3}$$

$$=\frac{3\times2+2\sqrt{6}+3\sqrt{6}+2\times3}{18-12}$$

$$=\frac{12+5\sqrt{6}}{6}=\frac{12}{6}+\frac{5\sqrt{6}}{6}=2+\frac{5\sqrt{6}}{6}$$

$$\therefore\ \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=a-b\sqrt{6}$$

$$\Rightarrow\ 2+\frac{5\sqrt{6}}{6}=a-b\sqrt{6}\ \Rightarrow\ a=2\ \text{and}\ b=\frac{-5}{6}$$

$$\therefore\ (a+b)^2=\left(2-\frac{5}{6}\right)^2=\left(\frac{12-5}{6}\right)^2=\left(\frac{7}{6}\right)^2=\frac{49}{36}$$

26. (a); Rationalising the denominator,

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

$$=\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}\times\frac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}\times\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}\times\frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}}$$

$$=\frac{7\sqrt{3}(\sqrt{10}-\sqrt{3})}{(\sqrt{10})^2-(\sqrt{3})^2}-\frac{2\sqrt{5}(\sqrt{6}-\sqrt{5})}{(\sqrt{6})^2-(\sqrt{5})^2}-\frac{3\sqrt{2}(\sqrt{15}-3\sqrt{2})}{(\sqrt{15})^2-(3\sqrt{2})^2}$$

$$=\frac{7\sqrt{3}(\sqrt{10}-\sqrt{3})}{10-3}-\frac{2\sqrt{5}(\sqrt{6}-\sqrt{5})}{6-5}-\frac{3\sqrt{2}(\sqrt{15}-3\sqrt{2})}{15-9\times2}$$

$$=\sqrt{3}(\sqrt{10}-\sqrt{3})-2\sqrt{5}(\sqrt{6}-\sqrt{5})+\sqrt{2}(\sqrt{15}-3\sqrt{2})$$

$$=\sqrt{30}-3-2\sqrt{30}+2\times5+\sqrt{30}-3\times2$$

$$=-3+10-6+\sqrt{30}-2\sqrt{30}+\sqrt{30}$$

$$=1+(1-2+1)\sqrt{30}=1$$

27. (b); $\frac{\sqrt{5}-2}{\sqrt{5}+2}-\frac{\sqrt{5}+2}{\sqrt{5}-2}=\frac{(\sqrt{5}-2)^2-(\sqrt{5}+2)^2}{(\sqrt{5}+2)(\sqrt{5}-2)}$

$$=\frac{\left[(\sqrt{5})^2+2^2-2\times\sqrt{5}\times2\right]-\left[(\sqrt{5})^2+2^2+2\times\sqrt{5}\times2\right]}{(\sqrt{5})^2-(2)^2}$$

$$=\frac{(5+4-4\sqrt{5})-(5+4+4\sqrt{5})}{5-4}$$

$$=9-4\sqrt{5}-9-4\sqrt{5}=-8\sqrt{5}$$

28. (c); Rationalising the denominator,

$$\frac{1}{3-\sqrt{8}}\times\frac{3+\sqrt{8}}{3+\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}\times\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}\times\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}+\frac{1}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}$$

$$=\frac{3+\sqrt{8}}{3^2-(\sqrt{8})^2}-\frac{\sqrt{8}+\sqrt{7}}{(\sqrt{8})^2-(\sqrt{7})^2}+\frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}-\frac{\sqrt{6}+\sqrt{5}}{(\sqrt{6})^2-(\sqrt{5})^2}+\frac{\sqrt{5}+2}{(\sqrt{5})^2-(2)^2}$$

$$=\frac{3+\sqrt{8}}{9-8}-\frac{\sqrt{8}+\sqrt{7}}{8-7}+\frac{\sqrt{7}+\sqrt{6}}{7-6}-\frac{\sqrt{6}+\sqrt{5}}{6-5}+\frac{\sqrt{5}+2}{5-4}$$

$$=(3+\sqrt{8})-(\sqrt{8}+\sqrt{7})+(\sqrt{7}+\sqrt{6})-(\sqrt{6}+\sqrt{5})+(\sqrt{5}+2)$$

$$=3+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2$$

$= 3 + 2 = 5$

29. (d); Expression $=(9)^{-3}\times\frac{(16)^{\frac{1}{4}}}{(6)^{-2}}\times\left(\frac{1}{27}\right)^{-\frac{4}{3}}$

$$=(3^2)^{-3}\times\frac{(2^4)^{\frac{1}{4}}}{(2\times3)^{-2}}\times\left(\frac{1}{3^3}\right)^{-\frac{4}{3}}$$

$$=3^{-6}\times\frac{2}{2^{-2}\times3^{-2}}\times(3^3)^{\frac{4}{3}}\qquad\left[\because\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n\right]$$

$$=3^{-6}\times2\times3^4\times2^2\times3^2\qquad\left[\because a^n=\frac{1}{a^{-n}}\right]$$