

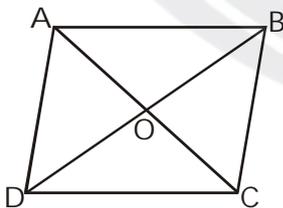
## Chapter - 3

## Quadrilaterals

### Foundation

#### Solutions

1. (b); Diagonals of parallelogram bisect each other.

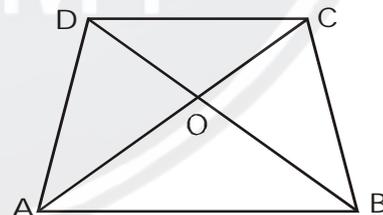


$$\therefore OC = OA = 3 \text{ cm}, \quad OD = OB = 4 \text{ cm}$$

$$AC = 2 \times OA = 6 \text{ cm} \quad BD = 2 \times OB = 8 \text{ cm}$$

2. (d);  $\angle APB = 90^\circ$  (because in parallelogram sum of two consecutive angle =  $180^\circ$  and angle bisector of  $(A+B) = 90^\circ$   
so, third vertex will be  $90^\circ$ )

3. (a); In  $\triangle AOB$  and  $\triangle COD$ ,



$$\angle OAB = \angle OCD$$

(alternate interior angle)

$$\angle OBA = \angle ODC$$

(alternate interior angle)

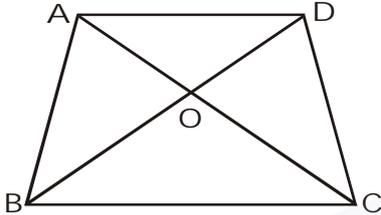
- $\therefore \triangle AOB \sim \triangle COD$   
(by AA - similarity)

$$\frac{\text{area}(\triangle AOB)}{\text{area}(\triangle COD)} = \frac{AB^2}{CD^2} = \frac{(2CD)^2}{CD^2} = \frac{4CD^2}{CD^2} = 4$$



$$\begin{aligned} \therefore \text{Area}(\Delta COD) &= \frac{1}{4} \times \text{area}(\Delta AOB) \\ &= \frac{1}{4} \times 84 = 21 \text{ cm}^2 \end{aligned}$$

4. (c); In  $\Delta AOB$  and  $\Delta BOC$ ,

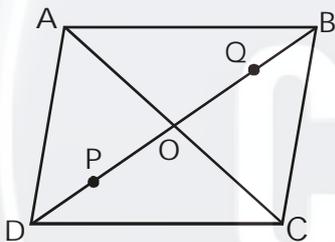


$$\frac{OA}{OC} = \frac{OB}{OB}$$

$$\frac{3}{x-3} = \frac{x-5}{3x-19}$$

$$\begin{aligned} \Rightarrow 3(3x-19) &= (x-3)(x-5) \\ \Rightarrow 9x-57 &= x^2-8x+15 \\ \Rightarrow x^2-17x+72 &= 0 \\ \therefore x &= 8, 9 \end{aligned}$$

5. (b); OB and OD are medians.



Diagonal divides parallelogram into two congruent triangles

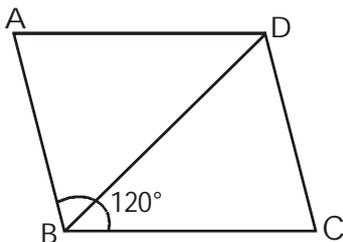
$$\frac{DP}{PO} = \frac{2}{1}, \frac{BQ}{OQ} = \frac{2}{1}$$

and  $PO = OQ$ .

$$\text{so, } PO = \frac{1}{3} \times 9 = 3$$

$$OQ = 3, \quad PO = OQ = 6$$

6. (a); Diagonals of rhombus bisect each angle.



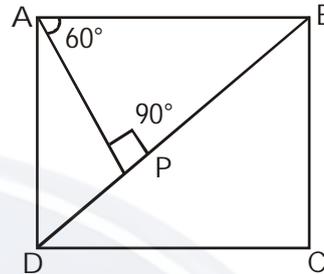
$$\therefore \angle ABD = \frac{120^\circ}{2} = 60^\circ$$

$$\begin{aligned} \therefore \angle ABD &= \angle ADB = 60^\circ \\ \therefore AB &= AD \\ \therefore \Delta ABD &\text{ will be equilateral triangle} \\ \therefore BD &= 4 \text{ cm} \end{aligned}$$

7. (a);

8. (c);

9. (c);

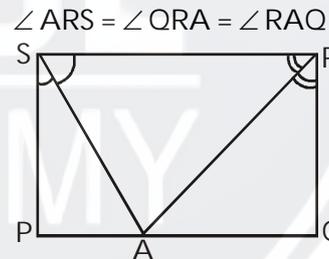


$$\begin{aligned} \angle PDC &= 90^\circ - \angle ADP \\ \angle ADP &= 180^\circ - \angle DAP - \angle APD \\ &= 180^\circ - 30^\circ - 90^\circ \\ \angle ADP &= 60^\circ \\ \angle PDC &= 90^\circ - \angle ADP = 90^\circ - 60^\circ \\ \angle PDC &= 30^\circ \end{aligned}$$

10. (b);

$$\begin{aligned} \angle B + \angle C &= 180^\circ \\ \angle C &= 180^\circ - 90^\circ = 90^\circ \\ \text{In quadrilateral AECD,} \\ \angle A + \angle E + \angle C + \angle D &= 360^\circ \\ \Rightarrow 90^\circ + (180^\circ - 60^\circ) + 90^\circ + \angle D &= 360^\circ \\ \therefore \angle D &= 60^\circ \end{aligned}$$

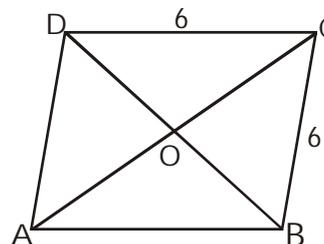
11. (b);



$$\begin{aligned} \therefore RQ &= AQ \\ \text{And } \angle PSA &= \angle ASR = \angle SAP \\ \therefore PS &= PA \\ \therefore PS + QR &= PA + AQ \\ \therefore 2QR &= SR \end{aligned}$$

12. (b);

$$\begin{aligned} AB = BC = CD = AD &= 6 \text{ cm} \\ \therefore ABCD &\text{ is also a rhombus. Diagonals of} \\ &\text{rhombus bisect each other at } 90^\circ. \end{aligned}$$



$\therefore OD = \frac{1}{2} BD = 3 \text{ cm}$

$\therefore OC = \sqrt{6^2 - 3^2} = \sqrt{27} = 3\sqrt{3} \text{ cm}$

$\therefore AC = 2OC$   
 $= 2 \times 3\sqrt{3} = 6\sqrt{3} \text{ cm}$

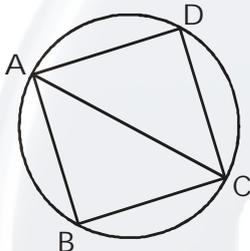
13. (a); Area of rhombus =  $\frac{1}{2}$  × product of diagonals

$96 = \frac{1}{2} \times 12 \times \text{second diagonal}$

$\therefore$  Second diagonal = 16 cm

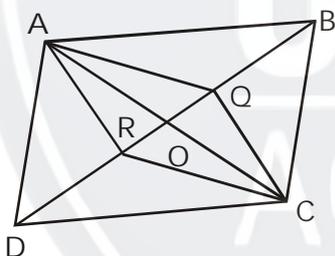
Length of side =  $\sqrt{\left(\frac{16}{2}\right)^2 + \left(\frac{12}{2}\right)^2} = \sqrt{8^2 + 6^2}$   
 $= 10 \text{ cm}$

14. (c); ABCD is a cyclic quadrilateral.



$\therefore \angle A + \angle C = \angle B + \angle D$

15. (b); Q is circumcentre of



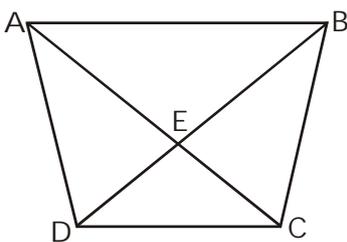
$\therefore$  QO is perpendicular of AC.

So, RO is also perpendicular of AC and  $\triangle ABC$  and  $\triangle ADC$  is congruent.

$\therefore$  AC and QR are perpendicular to each other

$\therefore$  AQCR is a rhombus.

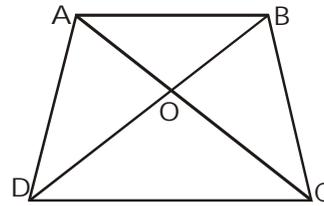
16. (d);  $\triangle DEC \sim \triangle BEA$



$\therefore \frac{DE}{EB} = \frac{EC}{EA}$

$\therefore DE \cdot EA = EB \cdot EC$

17. (a);  $\triangle AOB \sim \triangle COD$



$\therefore \frac{AO}{CO} = \frac{AB}{CD}$

$= \frac{10}{14} = \frac{5}{7} \Rightarrow 5 : 7$

18. (a);  $\angle A + b^\circ = 180^\circ$  (linear pair) ... (i)

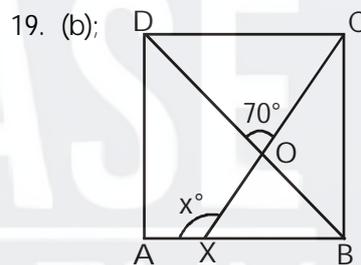
$\angle C + a^\circ = 180^\circ$  (linear pair) ... (ii)

From (i) and (ii)

$\angle A + \angle B + \angle C + \angle D = 360^\circ$

$180^\circ - b^\circ + x^\circ + 180^\circ - a^\circ + y^\circ = 360^\circ$

$x + y = a + b$



In  $\triangle XOB$ ,

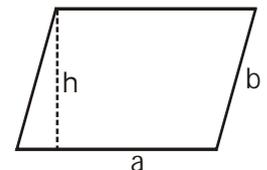
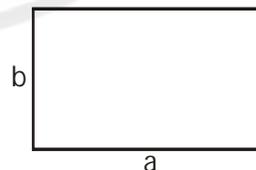
$\angle XOB + \angle BXO + \angle OBX = 180^\circ$

$70^\circ + \angle BXO + 45^\circ = 180^\circ$

$\angle BXO = 65^\circ$

$\angle OXA = 180^\circ - 65^\circ = 115^\circ$

20. (c); Let height of parallelogram = h



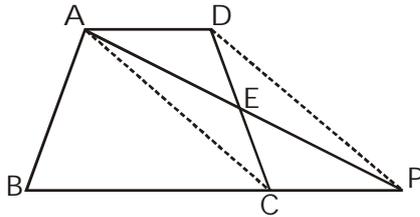
$h < b$

$A = a \times h, B = a \times b$

so,  $A < B$

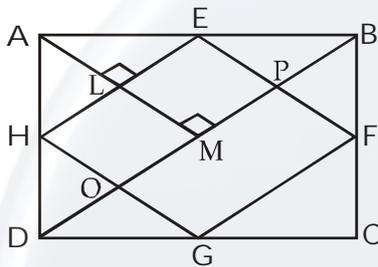
**Moderate**

1. (b);  $\therefore DP \parallel AC$ ,



$\therefore \text{ar}(\triangle ACD) = \text{ar}(\triangle ACP)$   
 Adding both side ar  $(\triangle ABC)$ ,  
 $\text{ar}(\triangle ABC) + \text{ar}(\triangle ACD)$   
 $= \text{ar}(\triangle ABC) + \text{ar}(\triangle ACP)$   
 $\therefore \text{ar}(ABCD) = \text{ar}(\triangle ABP)$

2. (C); Join B and D.



Draw  $AM \perp BD$ , which cuts HE at L.  
 In  $\triangle ABD$ ,  
 Midpoint of AB is E and midpoint of AD is H;

$\therefore HE \parallel DB$  and  $HE = \frac{1}{2} DB$   
 similarly,  $GF \parallel DB$   
 $\therefore EFGH$  is parallelogram  
 In  $\triangle ADM$ ,  
 Midpoint of AD is H and  $HL \parallel DM$

$\therefore AL = LM = \frac{1}{2} AM$

$$\text{ar}(\triangle ABD) = \frac{1}{2} \times BD \times AM$$

$$\text{ar}(\text{HEPO}) = EH \times LM = \frac{1}{2} BD \times \frac{1}{2} AM$$

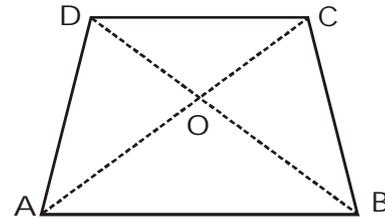
$$= \frac{1}{2} \left( \frac{1}{2} BD \times AM \right) = \frac{1}{2} \text{ar}(\triangle ABD)$$

similarly, area  $(\text{OPFG}) = \frac{1}{2} \text{ar}(\triangle CBD)$

$$\text{area of } (EFGH) = \frac{1}{2} \text{ar}(\triangle ABD) + \frac{1}{2} \text{ar}(\triangle CBD)$$

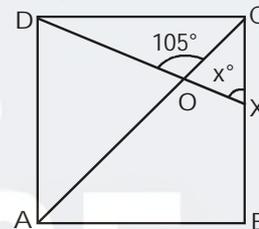
$$= \frac{1}{2} \text{ar}(ABCD)$$

3. (c); Join AC and BD.



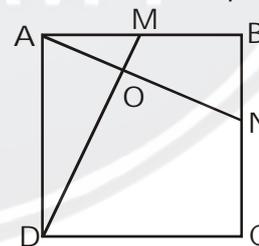
In  $\triangle ABC$ ,  
 $AB > BC$   
 $\therefore \angle ACB > \angle BAC$   
 In  $\triangle ADC$ ,  
 $AD > DC$   
 $\therefore \angle ACD > \angle CAD$   
 $\therefore \angle ACB + \angle ACD > \angle BAC + \angle CAD$   
 $\therefore \angle C > \angle A$

4. (c); Diagonal of a square bisects angle.



$\therefore \angle OCX = 45^\circ$   
 $(\because \angle DCB = 90^\circ$  and CA bisect  $\angle DCB)$   
 $\angle COD + \angle COX = 180^\circ$  (linear pair)  
 $105^\circ + \angle COX = 180^\circ, \angle COX = 75^\circ$   
 In  $\triangle COX$ ,  
 $\angle OCX + \angle COX + \angle OXC = 180^\circ$   
 $45^\circ + 75^\circ + \angle OXC = 180^\circ$   
 $\therefore \angle OXC = 60^\circ$

5. (b); Let each side of square = x



$\therefore AB = BC = CD = AD = x$

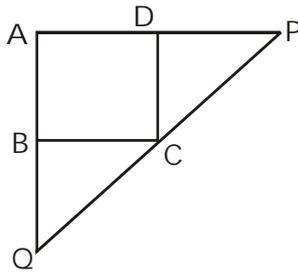
$$BN = AM = \frac{x}{2}$$

$$AN = \sqrt{x^2 + \left(\frac{x}{2}\right)^2} + \sqrt{x^2 + \frac{x^2}{4}} = \sqrt{\frac{5x^2}{4}}$$

$$DM = \sqrt{x^2 + \left(\frac{x}{2}\right)^2} = \sqrt{\frac{5x^2}{4}} \text{ so, } AN = DM$$

6. (a); Let each side of rhombus = x

Let BQ = a



∴ AB = BC = CD = AD = x

$$DP = \frac{1}{2} AB$$

$$\therefore DP = \frac{x}{2}$$

∆ APQ ~ ∆ DPC

$$\frac{AP}{DP} = \frac{AQ}{CD}, \quad \frac{x + \frac{x}{2}}{\frac{x}{2}} = \frac{x+a}{x}$$

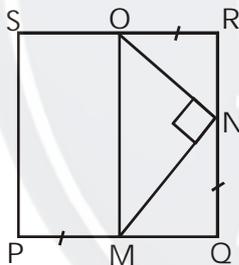
$$\Rightarrow 3x = x + a$$

$$\therefore a = 2x$$

$$\therefore BQ : AB$$

$$= a : x = 2x : x = 2 : 1$$

7. (b);



$$PM = QN$$

$$MQ = NR$$

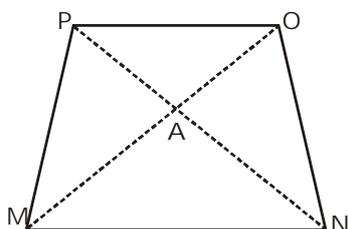
$$MN = NO$$

$$\therefore \angle NOM = \angle NMO$$

$$\therefore \angle NOM + \angle NMO = 90^\circ$$

$$\therefore \angle NOM = 45^\circ$$

8. (a); From ∆AMN and ∆APO



∠ OAP = ∠ NAM (Vertically opposite angle)

∠ ANM = ∠ APO (Alternate interior angle)

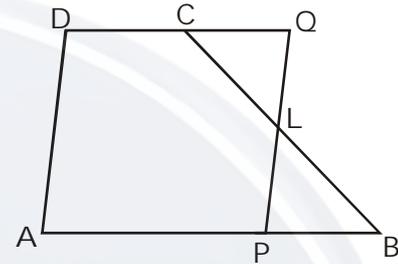
∠ AOP = ∠ AMN (Alternate interior angle)

∴ ∆ AMN ~ ∆ AOP

$$\Rightarrow \frac{AO}{AM} = \frac{AP}{AN} = \frac{OP}{MN}$$

$$\therefore \frac{AM}{AO} = \frac{AN}{AP} = \frac{MN}{OP}$$

9. (a); Form ∆ LPB and ∆ LQC,



$$BL = CL,$$

$$\angle BPL = \angle CQL$$

$$\angle PBL = \angle QCL$$

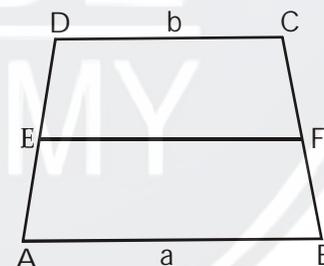
∴ ∆ LPB ≅ ∆ LQC

∴ ar (trapezium ABCD)

$$= \text{ar} (\text{APQD} + \triangle \text{PBL} - \triangle \text{QCL})$$

$$= (\text{Igm APQD})$$

$$10. (c); EF = \frac{1}{2} (a + b)$$



Let distance between AB and EF = d

∴ ar (trapezium ABEF)

$$= \frac{1}{2} \left( a + \frac{a+b}{2} \right) \times d = \frac{(3a+b)}{4} d$$

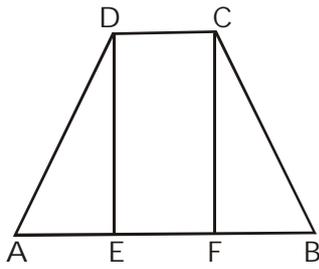
$$\text{ar (trapezium EFCD)} = \frac{1}{2} \left( \frac{a+b}{2} + b \right) \times d$$

$$= \frac{(a+3b)}{4} d$$

$$\therefore \text{ratio} = \frac{(3a+b)d}{4} : \frac{(a+3b)d}{4} = (3a+b) : (a+3b)$$



11. (d); In  $\triangle ABC$



$\angle B$  is acute angle.

$$AC^2 = BC^2 + AB^2 - 2 AB \cdot FB \quad \dots(i)$$

In  $\triangle ABD$ ,

$\angle A$  is acute angle.

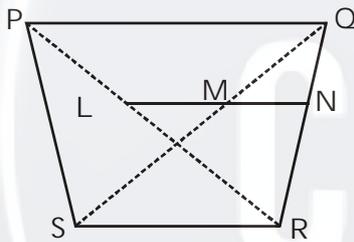
$$\therefore BD^2 = AD^2 + AB^2 - 2AB \cdot AE \quad \dots(ii)$$

From equation (i) + equation (ii),

$$(AC)^2 + (BD)^2 = BC^2 + AD^2 + 2AB(AB - FB - AE) = BC^2 + AD^2 + 2AB \cdot CD.$$

12. (d);  $PQ \parallel RS$

$$\therefore LM \parallel PQ \parallel RS$$



From  $\triangle PQR$ ,

$LN \parallel PQ$

$$\therefore LN = \frac{PQ}{2} \quad \dots(i)$$

From  $\triangle QRS$ ,

$MN \parallel RS$

$$\therefore MN = \frac{SR}{2} \quad \dots(ii)$$

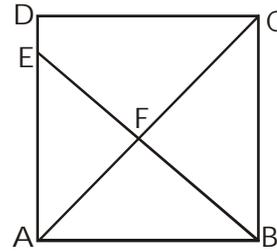
$$\Rightarrow LN - LM = \frac{SR}{2}$$

$$\Rightarrow \frac{QR}{2} - LM = \frac{SR}{2} \text{ [using (i)]}$$

$$\Rightarrow LM = \frac{QP}{2} - \frac{SR}{2}$$

$$\therefore LM = \frac{QP - SR}{2}$$

13. (b); In  $\triangle$  and  $\triangle CFB$ ,



$$\angle EFA = \angle CFB$$

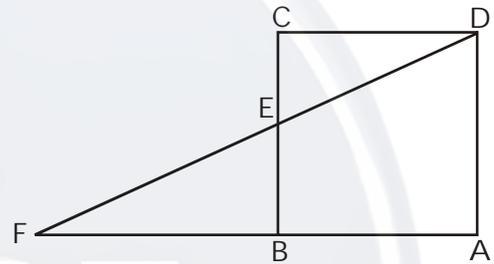
$$\angle EAF = \angle BCF$$

$$\angle AEF = \angle FCB$$

$$\therefore \frac{BF}{FE} = \frac{FC}{FA}$$

$$\therefore BF \cdot FA = FE \cdot FC$$

14. (c);  $\triangle BFE \cong \triangle CDE$



$$\left[ \begin{array}{l} \therefore \angle BFE = \angle CDE, \\ \angle BEF = \angle CED, CE = BE \end{array} \right]$$

$$\therefore FB = CD$$

But  $CD = BA$

$$\therefore FB = BA$$

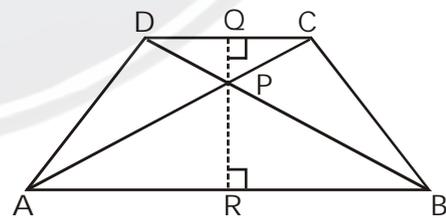
$$\therefore FB + BA = BA + BA$$

$$\Rightarrow FB + BA = 2BA$$

$$\therefore AF = 2AB$$

15. (d);  $AB \parallel CD$  and

$$AB = 2 CD$$



$\triangle APB \sim \triangle CPD$

$$\text{so, } \frac{PA}{PC} = \frac{PB}{PD} = \frac{AB}{DC} = \frac{PR}{QP}$$

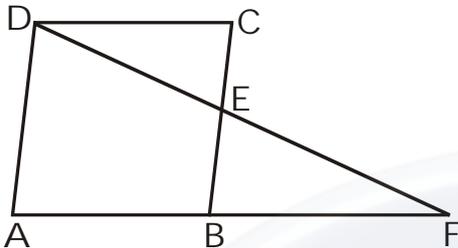
$$\frac{\text{ar}(\triangle APB)}{\text{ar}(\triangle CPD)} = \frac{\frac{1}{2} \times AB \times PR}{\frac{1}{2} \times CD \times QP}$$



$$= \frac{AB \times PR}{CD \times PQ} = \frac{AB}{CD} \times \frac{AB}{CD} \text{ [using (i)]}$$

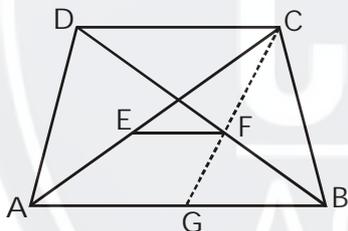
$$= \left(\frac{AB}{CD}\right)^2 = \left(\frac{2}{1}\right)^2 = 4 : 1$$

16. (b);  $\triangle EDC \cong \triangle EFB$



$\angle DEC = \angle BEF$   
 $\angle DCE = \angle EBF$  (Alternate angle)  
 $\triangle CDE \cong \triangle BFE$   
 $\frac{CD}{BF} = \frac{DE}{FE} = \frac{CE}{BE}$ ,  $\frac{CE}{BE} = \frac{CD}{BF}$   
 $CD = BF$  ( $CE = BE$ )  
 so,  $AF = AB + BF$   
 $= AB + CD$   
 $AF = 2AB \Rightarrow AF^2 = 4AB^2$

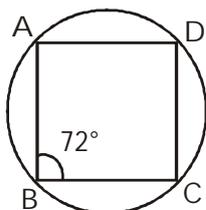
17. (d); Join CF and Produce at G on AB.



Then,  $\triangle CDF \cong \triangle GBF$   
 $CD = GB$  and  $CF = GF$   
 E and F are mid points of CA and CG.

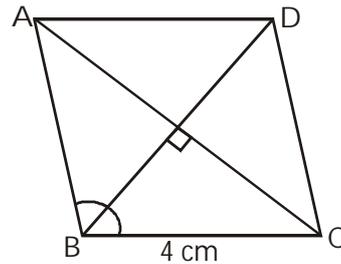
$$\therefore EF = \frac{1}{2} AG = \frac{1}{2} (AB - CD)$$

18. (d);  $\angle CDA = 180 - \angle ABC$



$= 180^\circ - 72^\circ = 108^\circ$   
 $AD \parallel BC$   
 $\angle BCD + \angle CDA = 180^\circ$   
 $\angle BCD = 180^\circ - 108^\circ = 72^\circ$

19. (d);  $\angle ABC = 120^\circ$



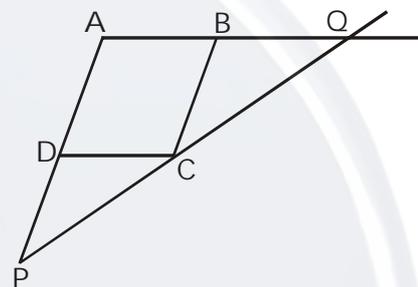
$$\angle DBC = \frac{120}{2} = 60^\circ$$

In  $\triangle BDC$ ,  $BC = CD$

$$\angle DBC = \angle BDC = 60^\circ$$

so,  $\triangle BDC$  is equilateral triangle,  $BD = 4 \text{ cm}$

20. (a); ABCD is rhombus.



$AB = BC = CD = AD$

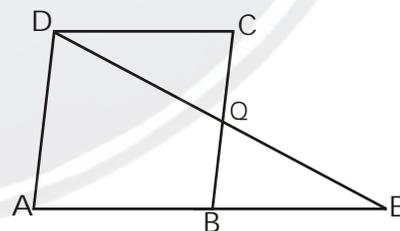
$$DP = \frac{1}{2} AB = \frac{1}{2} BC = \frac{1}{2} CD = \frac{1}{2} DA$$

from similarity

$$\frac{AB + BQ}{BQ} = \frac{AD + DP}{BC}$$

$$\frac{AB}{BQ} = \frac{3}{2} - 1 = \frac{1}{2}, \frac{BQ}{AB} = \frac{2}{1}$$

21. (b);  $AD \parallel BC$



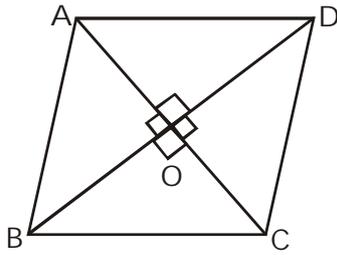
$AD \parallel BQ$

$$\triangle DQC \sim \triangle EQB \begin{cases} \angle DQC = \angle BQE \\ \angle DCQ = \angle QBE \\ \angle CDQ = \angle QEB \end{cases}$$

$$\frac{DQ}{QE} = \frac{CQ}{BQ} = \frac{CD}{BE} \Rightarrow \frac{CD}{BE} = \frac{CQ}{BQ}$$

$$CQ : BQ = 1 : 1$$

22. (b); In  $\triangle OBC$



$$OB^2 + OC^2 = BC^2$$

In  $\triangle OCD$ ,

$$OC^2 + OD^2 = CD^2$$

In  $\triangle OAD$ ,

$$OD^2 + OA^2 = AD^2$$

In  $\triangle OAB$ ,

$$OA^2 + OB^2 = AB^2$$

(i) + (ii) + (iii) + (iv)

$$2(OB^2 + OA^2 + OD^2 + OC^2)$$

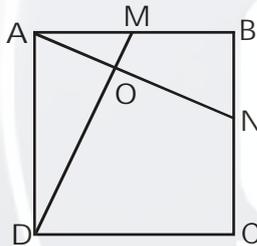
$$= AB^2 + BC^2 + CD^2 + DA^2$$

$$\Rightarrow 2(AB^2 + CD^2)$$

$$= AB^2 + BC^2 + CD^2 + DA^2$$

$$\Rightarrow AB^2 + CD^2 = BC^2 + DA^2$$

23. (b); Let  $BA = BC = CD = AD = x$  cm



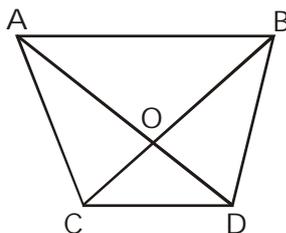
$$BN = NC = \frac{x}{2} \text{ cm}$$

In  $\triangle ABN$ ,

$$AN = \sqrt{x^2 + \left(\frac{x}{2}\right)^2}$$

$$= \frac{\sqrt{5}x}{2} \text{ cm}$$

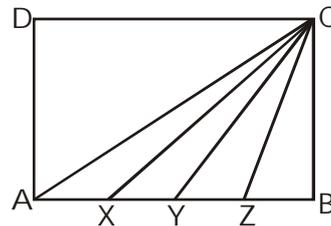
24. (b);  $\triangle OAB \sim \triangle OCD$



$$\frac{\text{area}(\triangle OAB)}{\text{area}(\triangle OCD)} = \frac{AB^2}{DC^2}$$

$$\frac{(2DC)^2}{DC^2} = \frac{4DC^2}{DC^2} = \frac{4}{1}$$

25. (d);  $AX = XY = YZ = BZ$



...(i)

...(ii)

...(iii)

...(iv)

Area of triangle on same base is equal in ratio of base.

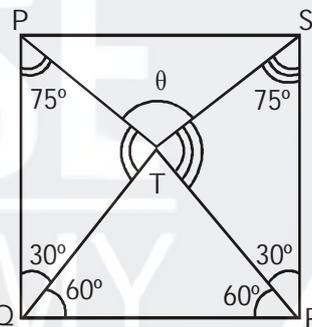
$$\text{area}(\triangle ACX) = \text{area}(\triangle XYC) = \text{area}(\triangle YZC) = \text{area}(\triangle BZC)$$

$$\text{area}(\triangle ABC) = \frac{1}{2} \text{area}(ABCD)$$

$$\text{area}(\triangle XYC) = \frac{1}{8} \text{area}(ABCD)$$

$$\frac{\text{Area}(\triangle XYC)}{\text{Area}(ABCD)} = \frac{1}{8}$$

26. (d); In square PQRS



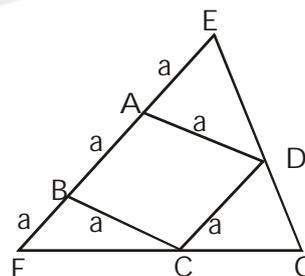
$\triangle PTS$

$$\angle SPT = 90^\circ - 75^\circ = 15$$

$$\angle PST = 90^\circ - 75^\circ = 15$$

$$\therefore \angle PTS = 180^\circ - (15 + 15) = 180^\circ - 30^\circ = 150^\circ$$

27. (c);  $AE = AD$  (In  $\triangle ADE$ )



$$\angle E = \angle D = a$$

$$\angle A = a + a = 2a, \quad \angle C = \angle F = a$$



$\angle B = d + d = 2d, \angle A + \angle B = 180^\circ$

$2a + 2a = 180^\circ, a = 45^\circ$

In  $\triangle OEF,$

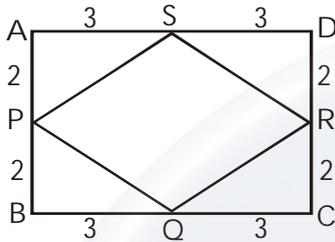
$\angle O + \angle E + \angle F = 180^\circ$

$\angle O + 45 + 45 = 180^\circ$

$\angle O = 90^\circ$

$ED \perp CF$

28. (d): Let sides of ABCD is-



$AD = BC = 6$  unit and

$AB = CD = 4$  unit

$\therefore$  Area ABCD =  $6 \times 4 = 24$  And

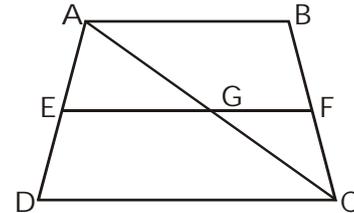
Area PQRS = ABCD - [ $\triangle SAP + \triangle PBQ + \triangle QCR + \triangle RDS$ ]

$= 24 - \frac{1}{2} [2 \times 3 + 2 \times 3 + 2 \times 3 + 2 \times 3]$

$= 24 - \frac{1}{2} \times 24 = 24 - 12 = 12$  unit

$\therefore$  Area PQRS =  $\frac{1}{2}$  area ABCD

29. (c); In a trapezium ABCD



$DC = p, AB = q$

$AB \parallel DC$

In  $\triangle ADC, EF \parallel DC$

$EG = \frac{1}{2} DC$  ... (i)

Similarly, In  $\triangle ABC,$

$AB \parallel GF$

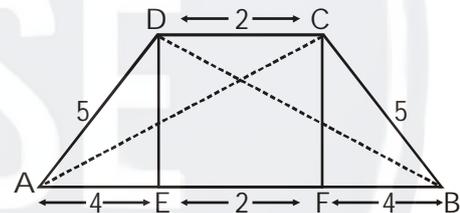
$GF = \frac{1}{2} AB$  ... (ii)

(i) + (ii)

$EG + GF = \frac{1}{2} DC + \frac{1}{2} AB$

$EF = \frac{1}{2} (DC + AB), EF = \frac{1}{2} (p + q)$

30. (a); In  $\triangle ACF$



$AC = \sqrt{6^2 + 3^2} = \sqrt{45} = 3\sqrt{5}$  cm

In  $\triangle BDE, DB = \sqrt{6^2 + 3^2} = 3\sqrt{5}$  cm

## Formulas of Quadrilateral

→ Perimeter Formulas

### 2. Perimeter of Quadrilaterals

→ In geometry, the perimeter can be defined as the path or the boundary that surrounds a shape. It can also be defined as the length of the outline of a shape.

Since we know that quadrilateral has four sides, therefore, the perimeter of any quadrilateral say, ABCD, is given by

<u>Perimeter of Parallelogram</u>	= $2 \times (\text{Base} + \text{Side})$
<u>Perimeter of Rectangle</u>	= $2 \times (\text{Length} + \text{Width})$
<u>Perimeter of Square</u>	= $4 \times \text{Side}$
<u>Perimeter of Rhombus</u>	= $4 \times \text{Side}$
<u>Perimeter of Trapezium</u>	= Sum of all Sides
<u>Perimeter of Kite</u>	= $2 \times (a + b)$ , where a, and b are Adjacent Pairs

