Chapter - 2

LCM and HCF

HCF and LCM Quick Maths Formulas

→ **Product of two numbers a and b (a*b) = Their HCF * Their LCM.**
But a*b*c ≠ HCF*LCM

#Note:
→ HCF of two or more numbers is the greatest number which divide all of them without any remainder.
→ LCM of two or more numbers is the smallest number which is divisible by all the given numbers.

→ **HCF of given fractions = (HCF of Numerator)/(LCM of Denominator)**
→ **HCF of given fractions = (LCM of Denominator)/(HCF of Numerator)**
→ **If d = HCF of a and b, then there exist unique integer m and n, such that -> d = am + bn.**

Foundation

Solutions

1. (a); Given, fractions $\frac{2}{3}, \frac{4}{9}$ and $\frac{5}{6}$

$$\text{LCM of fractions} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

$$= \frac{\text{LCM}(2,4,5)}{\text{HCF}(3,9,6)} = \frac{20}{3}$$

2. (d); HCF of two numbers is 8. This means 8 is a factor common to both the numbers.
∴ LCM must be multiple of 8. By going through the options, 60 cannot be the LCM since it is not a multiple of 8.

3. (a); HCF × LCM = Product of two numbers.
4× 520 = 52 × Second number

$$\therefore \quad \text{Second number} = \frac{4 \times 520}{52} = 40$$

4. (d); HCF × LCM
= First number × Second number

$\therefore$ Second number $= \frac{96 \times 1296}{864} = 144$

5. (d); HCF × LCM = Product of two numbers.
Then,

LCM $= \frac{216}{6} = 36$

6. (a); Let the numbers be 3x and 4x
HCF = x = 4
LCM of 3x and 4x = 12x = 12 × 4 = 48

7. (c); Given, first number = 132
Since, each of the two numbers is a multiple of 12 (given),
$\therefore$ HCF = 12 and LCM = 1056 (given)
LCM × HCF = First number × Second number

$\therefore$ Second number $= \frac{1056 \times 12}{132} = 96$

8. (d); Let the numbers be 3x and 4x respectively.
HCF×LCM
= First number × Second number
$\therefore$ 2028 = 3x × 4x

$x^2 = \frac{2028}{12} = 169$, x = 13

Numbers are = 39, 52
Sum of numbers = 39 + 52 = 91

9. (c); Let the numbers be 3x and 4x.
LCM of 3x and 4x = 12x
But given, LCM = 240
$\therefore$ 12x = 240

$\Rightarrow$ $x = \frac{240}{12} = 20$

$\therefore$ Smaller number = 3x = 3 × 20 = 60

10. (b); HCF × LCM = Product of two number

$\therefore$ Second number $= \frac{16 \times 160}{32} = 80$

11. (b); HCF × LCM = Product of two numbers

$\therefore$ LCM $= \frac{4107}{37} = 111$, ab × HCF = LCM

where a, b are prime factors $ab = \frac{111}{37} = 3$

Prime number pairs (3, 1)
Numbers = 3 × HCF, 1 × HCF
$\therefore$ Numbers are 111 and 37.
Hence, 111 is the greater number

12. (d); The ratio of two numbers is 3 : 4 and their HCF is 5. Their LCM is
Given that the numbers are in ratio of 3 : 4,
First number = 3 × 5 =15
Second number = 4 × 5 = 20
LCM × HCF = Product of two numbers

$\therefore$ LCM $= \frac{15 \times 20}{5} = 60$

13. (c); We know that,
LCM × HCF = Product of two numbers

$\therefore$ Second number $= \frac{26 \times 1820}{130} = 364$

14. (a); We know that,
HCF × LCM = Product of two numbers

$\therefore$ LCM = $= \frac{12906 \times 14818}{478}$

LCM = 400086

15. (b); Required number
= HCF of {(110 – 2) and (128 – 2)}
= HCF of 108 and 126
By Division Method

```
108)126(1
    108
    ---
     18)108(6
        108
        ---
         ×
```

$\therefore$ Greatest number = 18

16. (d); The LCM of 6, 12 and 18 = 36
36 is a perfect square of 6.

17. (a); The smallest number
= [LCM of (15, 20, 35) +k]

3	15, 20, 35
5	5, 20, 35
	1, 4, 7

LCM = 4 ×7 × 5 × 3= 420
Smallest number = 420 + 8 = 428

18. (a); 30 —— (5) × 3 × 2
35 —— (5) × 7
So, HCF = 5

19. (c); Let two numbers be 15x and 15y.Where x and y are co-prime to each other.
$\therefore$ 15x × 15y = 6300

$x \times y = \frac{6300}{15 \times 15}$

x × y = 28
Factors of 28 are 1, 2, 4, 7, 28
$\therefore$ 28 = 1 × 28 or 4 × 7
$\therefore$ There are only two possible pairs

20. (a); HCF + LCM = 403
HCF + 12 HCF = 403
(It is given that LCM = 12 × HCF)
13 × HCF = 403
HCF = 31
∵ Product of two number is equal to product of their LCM and their HCF.
∴ 93 × other number = 31 × 12 × 31
Other number = $\frac{31\times12\times31}{93}=124$

21. (b); We need to find the LCM of 12, 15, 20, 27.
$12 = 2^2 \times 3$
$15 = 3 \times 5$
$20 = 2^2 \times 5$
$27 = 3^3$
LCM = Product of highest powers of factors
$= 2^2 \times 3^3 \times 5 = 540$

22. (a); Let x be the remainder then (25 – x), (73 – x) and (97 – x) will be exactly divisible by the required number.
Required number = HCF of (73 – x)– (25 – x), (97 – x) – (73 – x) and (97 – x) – (25 – x)
= HCF of (73 – 25), (97 – 73) and (97 – 25)
= HCF of 48, 24 and 72 = 24

23. (d); LCM of 12, 15, 18, 27 = 540
Largest number of 5 digits = 99999
On dividing 99999 by 540, remainder = 99
∴ Required number = 99999 – 99 = 99900

24. (b); 20 – 14 = 25 – 19 = 35 – 29 = 40 – 34 = 6
Required number = (LCM of 20, 25, 35, 40) – 6
= 1400 – 6= 1394

25. (a); To find the biggest measure, we have to find the HCF of 496, 403 and 713.
HCF of 496, 403 and 713 = 31

26. (b); Required time = LCM of 20, 30 and 40

10	20, 30, 40
2	2, 3, 4
	1, 3, 2

LCM = 10 × 2 × 3 × 2 = 120
Hence, the bells will simultaneously ring after 2h i.e., at 1 pm

27. (a); Required time = LCM of (200, 300, 360, 450) s.

10	200, 300, 360, 450
5	20, 30, 36, 45
3	4, 6, 36, 9
2	4, 2, 12, 3
2	2, 1, 6, 3
3	1, 1, 3, 3
	1, 1, 1, 1

∴ LCM = 10 × 5 × 3 × 2 × 2 × 3 = 1800 s

28. (c); Let the numbers be ax and bx, where x is the HCF and bx > ax.
∴ LCM = abx
abx = 2bx
⇒ a = 2
Again, ax – x = 4
Putting the value of a, we get
2x – x = 4
⇒ x = 4
∴ Smaller number = ax = 2 × 4 = 8

29. (a); All the 6 bells ring together will be LCM of (2, 4, 6, 8, 10 and 12)

2	2, 4, 6, 8, 10, 12
2	1, 2, 3, 4, 5, 6
3	1, 1, 3, 2, 5, 3
	1, 1, 1, 2, 5, 1

∴ They will ring together after
= 2 × 2 × 2 × 3 × 5 = 120 s
i.e., they will ring together after 2 min
∴ Number of time they will ring together in 30 min = $1+\frac{30}{2}=1+15=16$ times

30. (c); Smallest number of boxes for buns alone
$=\frac{\text{LCM of 10 and 8}}{\text{Number of buns in a box}}=\frac{40}{10}=4$

Moderate

1. (a); Capacity of three containers containing diesel is 403 l, 434 l and 465 l, respectively.

Now, maximum capacity of the container that can measure the diesel of the three containers exactly

= HCF of quantity of three containers

= HCF (403, 434, 465)

```
403)434(1
    403
    ---
     31)403(13
        403
        ---
         ×
```

```
Again, 31)465(15
          465
          ---
           ×
```

So, Capacity of container = 31 L

2. (b); Time taken by A $= \frac{30}{5}$ h

Time taken by B $= \frac{30}{7}$ h

Time taken by C $= \frac{30}{8}$ h

Time taken by all to come together

$= \text{LCM of } \frac{30}{5}, \frac{30}{7} \text{ and } \frac{30}{8}$

$= \frac{\text{LCM of } 30, 30 \text{ and } 30}{\text{HCF of } 5, 7 \text{ and } 8} = \frac{30}{1} = 30\text{h}$

3. (b); LCM of 24, 36 and 54

2	24, 36, 54
2	12, 18, 27
2	6, 9, 27
3	3, 9, 27
3	1, 3, 9
3	1, 1, 3
	1, 1, 1

Required time = LCM of 24, 36 and 54 s = 216 s

$= \frac{216}{60} = 3\frac{36}{60}$ min

= 3 min 36 s

∴ Time when they will change simultaneously

= 10 : 15 : 00 + 3 min 36 s

= 10 : 18 : 36 am

4. (c); Let the numbers be 48a and 48b, where a and b are coprimes,

∴ 48a + 48b = 384

⇒ 48 (a +b) = 384

⇒ $a + b = \frac{384}{48} = 8$ ... (i)

Possible valid pairs of a and b satisfying this condition are (1, 7) and (3, 5).

∴ Numbers are 48 × 1 = 48

and 48 ×7 = 336 or

or, 48 × 3 = 144

and 48 × 5 = 240

∴ Required difference = 336 – 48 = 288

or 240 – 144 = 96

5. (c); Let the numbers be a and b.

According to the question,

a + b = 45 ... (i)

Again, $a - b = \frac{1}{9}(a + b)$

⇒ $a - b = \frac{1}{9} \times 45$ ⇒ a – b = 5 ... (ii)

Adding equ. (i) and (ii), we get

2a = 50

$a = \frac{50}{2} = 25$

On putting the value of a Eq. (i) we get

25 + b= 45

⇒ b = 45 – 25 = 20,

a = 25 and b = 20

∴ LCM of 25 and 20.

5	25, 20
5	5, 4
	1, 4

LCM = 5 ×5 × 4 = 100

6. (c); Let the numbers be 4a and 4b where a and b are coprimes.

LCM = 4ab

∴ $\frac{(4a + 4b)}{4ab} = \frac{7}{12}$

⇒ $\frac{1}{a} + \frac{1}{b} = \frac{7}{12} = \frac{1}{3} + \frac{1}{4}$

⇒ a = 3, b = 4

∴ First number = (4 × 3) = 12

Second number = (4 × 4) = 16

∴ Smaller number = 12

7. (b); Minimum number of students = LCM of 6, 8, 10.

2	6, 8, 10
2	3, 4, 5
2	3, 2, 5
	3, 1, 5

LCM = 2 × 2 × 2 × 3 × 5 = 120
Hence, the minimum number of students = 120

8. (a); Let the number be 2x, 3x and 4x, respectively.
∴ HCF = x = 12
∴ Numbers 2 × 12 = 24, 3 × 12 = 36, 4 × 12 = 48
LCM of 24, 36, 48

2	24, 36, 48
2	12, 18, 24
2	6, 9, 12
2	3, 9, 6
3	3, 9, 3
3	1, 3, 1
	1, 1, 1

∴ LCM = 2 × 2 × 2 × 2 × 3 × 3 = 144

9. (d); Given,

$3^{3^{333}}+1$ and $3^{3^{334}}+1$ or $27^{333}+1^{333}$ and $27^{334}+1^{334}$

Now, $x^m + a^m$ is divisible by (x + a) when m is odd.
$27^{333} + 1^{333}$ is divisible by (27 + 1) = 28
Similarly, $27^{334} + 1^{334}$ is never divisible by (x + a)
So, the greatest common divisor between $\left(3^{3^{333}}+1\right)$ and $\left(3^{3^{334}}+1\right)$ is 1.

10. (b); Maximum quantity in each can
= HCF of (21, 42 and 63) L = 21 L
By Division Method

21) 42 (2
42
×
21) 63 (3
63
×

HCF = 21 L
∴ Least number of cans

$=\frac{21}{21}+\frac{42}{21}+\frac{63}{21}$ = 1 + 2 + 3 = 6 cans.

11. (c); To find the minimum number of rows, we determine the HCF of 24, 36 and 60.
∴ HCF of 24, 36 and 60 = 12
Thus, 12 fruits are there in a row.

∴ Number of rows $=\frac{24}{12}+\frac{36}{12}+\frac{60}{12}$
= 2 + 3 + 5 = 10

12. (c); Let the numbers be p, q and r which are coprime to one another.
Now, pq = 551 and qr = 1073
q = HCF of 551 and 1073

551) 1073 (1
551
522) 551 (1
522
29) 522 (18
522
×

∴ q = 29 ∴ $p=\frac{551}{29}=19$

and $r=\frac{1073}{29}=37$

∴ Sum of three numbers.
= 19 + 29 + 37 = 85

13. (a); Number of books in each stack = HCF of (336, 240, 96)

240) 336 (1
240
96) 240 (2
192
48) 96 (2
96
×

∴ Number of books in each stack = 48

∴ Total number of stacks $=\frac{336}{48}+\frac{240}{48}+\frac{96}{48}$
= 7 + 5 + 2 = 14

14. (c); Givn, HCF = 3, LCM = 105
Now, let the numbers be 3a and 3b,
∴ 3a + 3b = 36
⇒ a + b = 12 ... (i)
and LCM 3ab = 105 ... (ii)
Dividing Eq. (i) by Eq. (ii), we have

$$\frac{a}{3ab}+\frac{b}{3ab}=\frac{12}{105}$$

$$\Rightarrow \frac{1}{3a}+\frac{1}{3b}=\frac{4}{35} \Rightarrow \frac{1}{a}+\frac{1}{b}=\frac{12}{35}$$

15. (d); Length of the floor = 15 m 17 cm = 1517 cm
Breadth of the floor = 9 m 2 cm = 902 cm
Area of the floor = 1517 × 902 cm^2
The number of square tiles will be least, when the size of each tile is maximum.
∴ Size of each tile = HCF of 1517 and 902

```
902)1517(1
    902
    615)902(1
        615
        287)615(2
            574
             41)287(7
                287
                 ×
```

HCF = 41

∴ Required number of tiles

$= \frac{\text{Area of the floor}}{\text{Area of each square tile}} = \frac{1517 \times 902}{41 \times 41} = 814$

16. (d); Given, HCF of a and b = 12
Let the numbers be 12x and 12y, where x and y are coprime.
But given a > b > 12
Smallest coprime pair for the above condition = (3, 2)
∴ a = 36 and b = 24

17. (b); Given,
$P = 2^3 \times 3^{10} \times 5$, $Q = 2^5 \times 3 \times 7$
[∵ 2^3 and 3 is common factor to both P and Q]
HCF = $2^3 \times 3$

18. (b); HCF = 12
Numbers = 12a and 12b where, a and b are coprimes
∴ 12a + 12b = 84 ⇒ 12 (a + b) = 84

$\Rightarrow a + b = \frac{84}{12} = 7$

∴ Possible pairs of numbers satisfying this condition = (1, 6), (2, 5) and (3, 4)
Hence, pairs of required numbers = 3.

19. (c); HCF of two numbers = 4
Hence, the numbers can be expressed as 4a and 4b, where a and b are coprime,
4a + 4b = 36, a + b = 9
Now, possible pairs satisfying above condition are (1, 8), (4, 5), (2, 7).
∴ 3 pairs are possible

20. (a); We know that,
HCF × LCM = Product of two numbers

$\therefore \text{LCM} = \frac{12908 \times 14808}{672} = 284437$

21. (c); Factors of 50 = $5^2 \times 2$
Factors of 80 = $5^1 \times 2^4$
H.C.F. of 50 & 80 = $5^1 \times 2^1$ = 10 l
The capacity of the mug must be 10 l

22. (a); LCM of 48, 72 and 108 = 432
The traffic lights will change simultaneously after 432 seconds or 7 m 12 secs.
∴ They will change simultaneously at
= 8 : 20 hours + 7 m + 12 sec. = 8 : 27 : 12 hrs.

23. (b); Product of two numbers = H.C.F. × L.C.M.
7 × 30 × Second number = 30 × 2310

$\text{Second number} = \frac{30 \times 2310}{7 \times 30} = 330$

24. (b); LCM of 2, 3, 4, 5 and 6 = 60
Other numbers divisible by 2, 3, 4, 5, 6 are 60k, where k is a positive integer. Since 2 – 1 = 1, 3 – 2 = 1, 4 – 3 = 1, 5 – 4 = 1 and 6 – 5 = 1, the remainder in each case is less than the divisor by 1. Now, the required number is to be divisible by 7. Hence, we must choose the least value of k which will make (60k – 1) divisible by 7. Putting k equal to 1, 2, 3 etc. in succession, we find that k should be 2
∴ The required number = 60k – 1
= 60 × 2 – 1 = 119

25. (d); Required number
= (LCM of 12, 15, 20 & 54) + 4
= 540 + 4 = 544

26. (a); Required number of students = HCF of 1001 and 910

```
910)1001(1
    910
     91)910(10
        910
          0
```

Hence, HCF = 91

27. (d); Required time = LCM of 252, 308 and 198 s.

2	252, 308, 198
2	126, 154, 99
7	63, 77, 99
9	9, 11, 99
11	1, 11, 11
	1, 1, 1

∴ LCM = 2 × 2 × 7 × 9 × 11

$= 2772 \text{ s} = \frac{2772}{60}$

$= 46\frac{1}{5} \text{ min} = 46 \text{ min } 12 \text{ s}$

28. (c); Minimum distance each should cover, so that all can cover the distance in complete steps
= LCM of (63, 70, 77) = 6930 cm

29. (b); LCM of 15, 18, 21, 24

2	15, 18, 21, 24
2	15, 9, 21, 12
2	15, 9, 21, 6
3	15, 9, 21, 3
3	5, 3, 7, 1
5	5, 1, 7, 1
7	1, 1, 7, 1
	1, 1, 1, 1

LCM $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2520$

Largest number of four digit = 9999

$$\begin{array}{r} 2520\overline{)9999}(3 \\ 7560 \\ \hline 2439 \end{array}$$

Required number = 9999 – 2439 – 4 = 7556

$$\text{Where, } 4 = \begin{Bmatrix} 15 - 11 = 4 \text{ or} \\ 18 - 14 = 4 \text{ or} \\ 21 - 17 = 4 \text{ or} \\ 24 - 20 = 4 \end{Bmatrix}$$

30. (b); LCM of 9 and 6 = 18

Total numbers from 1 to 200 divisible by 18 = 11

Total numbers from 1 to 100 divisible by 18 = 5

$\therefore$ Required numbers from 100 to 200 divisible by 18 = 11 – 5 = 6

HCF and LCM Tips and Tricks and Shortcuts

Some important HCF and LCM Rules

Factors and Multiples

→ If number a, divided another number b exactly, we say that a is a factor of b.
→ In this case, b is called a multiple of a Co-primes.
→ Two numbers are said to be co-primes if their H.C.F. is 1.
→ HCF of a given number always divides its LCM.

Tips and Tricks and Shortcuts

→ The H.C.F of two or more numbers is smaller than or equal to the smallest number of given numbers
→ The smallest number which is exactly divisible by a, b and c is L.C.M of a, b, c.
→ The L.C.M of two or more numbers is greater than or equal to the greatest number of given numbers.
→ The smallest number which when divided by a, b and c leaves a remainder R in each case. Required number = (L.C.M of a, b, c) + R
→ The greatest number which divides a, b and c to leave the remainder R is H.C.F of (a – R), (b – R) and (c – R)
→ The greatest number which divide x, y, z to leave remainders a, b, c is H.C.F of (x – a), (y – b) and (z – c)
→ The smallest number which when divided by x, y and z leaves remainder of a, b, c (x – a), (y – b), (z – c) are multiples of M
→ Required number = (L.C.M of x, y and z) – M