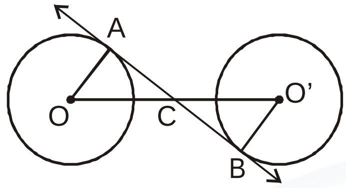
Chapter - 2

Circle

Foundation

Solutions

1. (c); The transverse common tangent to two equal circles bisects the line joining their centres.



$OO' = 10$ cm

$OA = O'B = 3$ cm

$AB = \sqrt{10^2 - 6^2} = 8$ cm

2. (d); At most four common tangents can be drawn to two circles.

3. (b); BD = BF (tangent through a common point)

CE = CF (tangent through a common point)

AD = AE (tangent through a common point)

$AD = AB + BD = AB + BF$ ...(i)

$AE = AC + CF$ ...(ii)

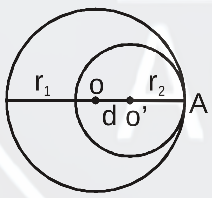
On adding (i) and (ii)

$AD + AE = AB + BC + CA$

$2AD = AB + BC + CA$

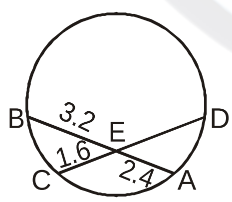
4. (c); Let O and O' be the centres of the two circles of radii r_1 and r_2 touching internally at A such that $OO' = d$.

Then $OA - O'A = OO' \Rightarrow r_1 - r_2 = d$



5. (a);

6. (c);



$AE \times BE = CE \times DE, \quad 3.2 \times 2.4 = 1.6 \times DE$

$DE = \frac{3.2 \times 2.4}{1.6} = 4.8$ cm

7. (c); $\pi r^2 = 200$

$r^2 = \frac{200}{\pi}, \quad r = \sqrt{\frac{200}{\pi}}, \quad R = 2r = 2\sqrt{\frac{200}{\pi}}$

Area of langer circle $= \pi R^2$

$= \pi \times 4 \times \frac{200}{\pi} = 800 \text{ cm}^2$

8. (b); Diameter of circle = 10 cm

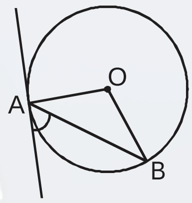
Diagonal of square = 10 cm

$\text{Side} = \frac{\text{diagonal}}{\sqrt{2}} = \frac{10}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$ cm

9. (b); The focus of the tangent is clearly the equation of the tangent, which is $xx_1 + yy_1 = a^2$.

10. (a); Chords equidistant from the centre of a circle are equal.

11. (c);



$\angle OAB = 90° - \theta$

$OA = OB$

$\angle OAB = \angle ABO = 90° - \theta$

12. (c); Any cyclic parallelogram having equal adjacent sides is necessarily a rhombus.

13. (a); If a regular hexagon is inscribed in a circle of radius r, then its perimeter is 6r.

14. (b); Radius of circle $= \frac{1}{2} \times$ Diagonal of square

$= \frac{1}{2} \times 4\sqrt{2} = 2\sqrt{2}$ cm

15. (a); Let the length of chord AB is x

then $PQ^2 = AQ \cdot BQ$

$12^2 = (x + BQ) \times BQ$

$144 = (x + 8)8$

$x + 8 = 18, \quad x = 10$ cm

16. (c); $\angle BAD = \angle BCD$ (Angle made by same arc BC)

$\angle BDC = 38°$

$OD = OC = r$ (radius of circle)

$\angle BDC = \angle OCD$

$\angle OCD = 38°$

17. (b); $OB = OC \Rightarrow \angle OBC = \angle OCB = 20°$

$\angle BOC = 180° - 40° = 140°$

$\angle BAC = \frac{1}{2}\angle BOC, \quad \angle BAC = \frac{1}{2} \times 140°$

$\angle BAC = 70°$

18. (d); $\angle ACB = \angle ADB = 20°$
$\angle ACB = \angle 20°$
$\angle CAB + \angle ACB + \angle ABC = 180°$
$85° + 20° + x = 180°$
$x = 180° - 105° = 75°$

19. (c); $\angle APB = \frac{1}{2}\angle AOB$

$\angle APB = \frac{1}{2} \times 90° = 45°$

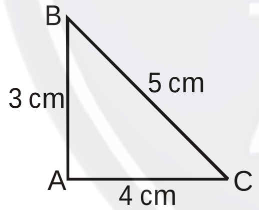
20. (d); $AP \times CP = PD \times PB$
$14 \times 9 = 7 \times (7 + x)$
$18 = 7 + x, \quad x = 11$ cm

21. (a); $x = 40°$ (Angle made by same arc AB.)

22. (b); $PT^2 = PA \cdot PB$
$12^2 = x \cdot (x + 7)$
$144 = x^2 + 7x$
$x^2 + 7x - 144 = 0$
$x^2 + 16x - 9x - 144 = 0$
$x(x + 16) - 9(x + 16) = 0$
$(x - 9)(x + 16) = 0$
$x = 9$ cm

23. (d); $PA = PB$
$\therefore \angle PAB = \angle PBA$
Also, $\angle PAB + \angle PBA + \angle APB = 180°$
$2\angle PAB = 120°$
$\angle PAB = 60°$
Therefore ΔPAB is an equilateral triangle.
$AB = AP = 6$ cm
$AB = 6$ cm

24. (a);



Area of $\Delta ABC = \frac{1}{2} \times 3 \times 4 = 6$

Semiperimeter $= \frac{3+5+4}{2} = 6$

Inradius $= \frac{\text{Area}}{\text{Semiperimeter}} = \frac{6}{6} = 1$ cm

25. (b); $\angle AOB = \angle COD = 70°$ (Vertically opposite angle)
$\angle COD + \angle OCD + \angle ODC = 180°$
$2\angle OCD = 180° - 70°$
$2\angle OCD = 110°$

$\angle OCD = \frac{110°}{2} = 55°$

26. (c);

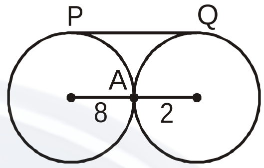
27. (c); $\angle CBA = \frac{1}{2}\angle COA = \frac{1}{2} \times 130° = 65°$

$\angle CBE = 180° - 65° = 115°$

28. (b); $\angle AOB + \angle APB = 180°$

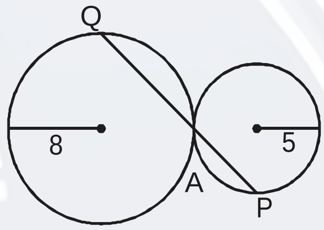
$\angle APB = \frac{1}{6} \times 180° = 30°$

29. (d);



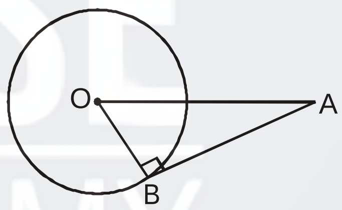
$PQ = 2\sqrt{r_1 r_2} = 2\sqrt{8 \times 2} = 2 \times 4 \text{ cm} = 8$ cm

30. (b);



Then the ratio $\frac{AP}{AQ} = \frac{5}{8} = 5:8$

31. (a);

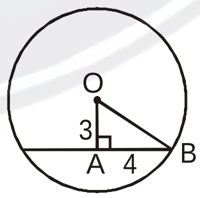


length of tangent

$AB = \sqrt{OA^2 - OB^2} = \sqrt{10^2 - 6^2}$

$= \sqrt{100 - 36} = \sqrt{64} = 8$ cm

32. (b);



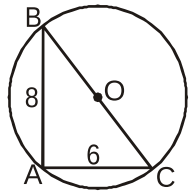
In ΔOAB
$OB^2 = OA^2 + AB^2$

$OB = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ cm

33. (b); $OB^2 = \sqrt{OA^2 - AB^2}$

$= \sqrt{10^2 - 8^2} = \sqrt{100 - 64} = \sqrt{36} = 6$

34. (d);



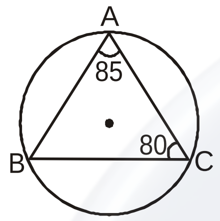
$\angle BAC = 90°$

BC is the diameter of the circle

$BC = \sqrt{AB^2 + AC^2} = \sqrt{8^2 + 6^2} = 10$ cm

Radius of circle = 5 cm

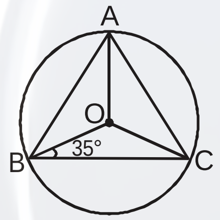
35. (b);



$\angle ABC = 180° - 80° - 85° = 15°$

$\Rightarrow \angle AOC = 2\angle ABC = 2 \times 15° = 30°$

36. (a);



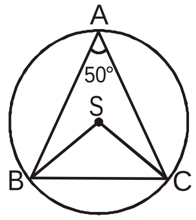
OB = OC = radius

$\angle OBC = \angle OCB = 35°$

$\angle BOC = 180° - 70° = 110°$

$\angle BAC = \frac{1}{2}\angle BOC = \frac{1}{2} \times 110 = 55°$

37. (b);



$\angle BAC = 50°$, $\angle BSC = 100°$

BS = SC = radius

$\angle BCS = \frac{1}{2}(180 - 100) = 40°$

38. (d); Transverse Common tangent

$= \sqrt{(\text{Distance between centres}) - (r_1 + r_2)^2}$

$= \sqrt{10^2 - (4+4)^2} = \sqrt{10^2 - 8^2} = \sqrt{36} = 6$ cm

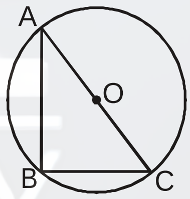
39. (d); Only one circle can pass through the non-collinear points.

40. (b); $3^2 + 4^2 = 5^2$

Therefore, ΔABC is a right angled triangle $\angle B = 90°$ = angle at the circumference

Diameter of circle = 5 cm

$\text{radius} = \frac{5}{2} = 2.5$ cm

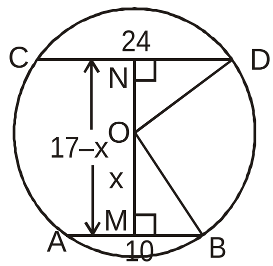


Moderate

1. (c); $\angle KLN = 30°$ (Given)
$\angle LKN = 90°$ (angle made in semicircle)
$\angle KNL = 180° - 30° - 90°$
$\angle KNL = 60°$
$\angle KNL = \angle PKL$ (angle in alternate segment)
$\angle PKL = 60°$

2. (a); $\angle ADC = 120°$
$\angle ABC = 180° - 120°$
$\angle ABC = 60°$
In ΔABC
$\angle ABC = 60°$, $\angle ACB = 90°$
(angle is semicircle)
$\angle BAC = 180° - 60° - 90° = 30°$

3. (d); Let OM = x
In ΔOMB
$x^2 + 5^2 = r^2$ ... (i)
In ΔOND
$(17 - x)^2 + 12^2 = r^2$
$x^2 + 5^2 = (17 - x)^2 + 12^2$
$34x = 408$

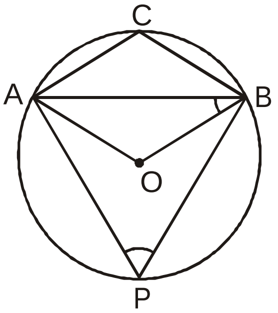


$\Rightarrow x = \frac{408}{34} = 12$ cm

From (i)

$12^2 + 5^2 = r^2$, $r = \sqrt{169} = 13$ cm

4. (c); OA = OB = radius



$\angle OAB = \angle OBA = 25°$

$\angle AOB = 180° - 25° - 25° = 130°$

$\angle APB = \frac{130°}{2} = 65°$, $\angle ACB = 180° - 65° = 115°$

5. (b); $\angle PQO = 90°$, $\angle POR = 120°$

$\angle POQ = 180° - 120° = 60°$

In ΔQPO

$\angle OQP + \angle QOP + \angle QPO = 180°$

$\angle QPO = 180° - 90° - 60° = 30°$

6. (d); OB = OC = OA = radius

$\angle OBA = \angle OAB$, $\angle OAB = 20°$

Similarly $\angle OAC = 30°$

$\angle BAC = 20° + 30° = 50°$

$\angle BOC = 2 \times 50° = 100°$

7. (c); $\angle ABC = 80°$

$\angle ADC = 180° - 80° = 100°$

$\angle DAB = 60°$

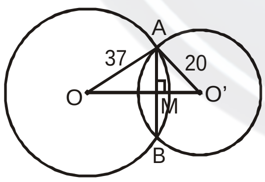
In ΔADQ

$\angle ADQ + \angle DAQ + \angle AQD = 180°$

$\angle AQD = 180° - 100° - 60°$

$\angle AQD = 20°$, $\angle BQC = 20°$

8. (b);



AM = 12 cm

In $\Delta AMO'$, $O'M^2 = O'A^2 - AM^2$

$O'M^2 = 20^2 - 12^2$

$O'M = 16$

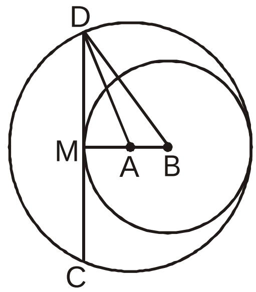
In AOM $OM^2 = 37^2 - 12^2$

$OM = 35$

then, $OO' = OM + MO'$

$= 35 + 16 = 51$ cm

9. (a); In ΔDAM



$AD^2 = AM^2 + DM^2$

$6^2 = 2^2 + DM^2$

$DM^2 = 6^2 - 2^2$

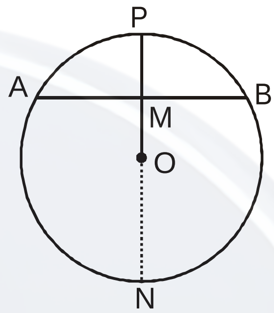
$DM^2 = 36 - 4$

$DM = \sqrt{32} = 4\sqrt{2}$

$CD = 2 \times DM$

$= 2 \times 4\sqrt{2} = 8\sqrt{2}$ cm

10. (b);



AB = 8 cm, MP = 2 cm

AM = 4 = MB, $AM \times MB = PM \times MN$

$4 \times 4 = 2 \times MN$, MN = 8 cm

MN = 2r – 2 = 8, 2r = 10, r = 5 cm

11. (a); $\angle ADC + \angle ABC = 180°$

$\angle ABC = 180° - 130° = 50°$

In ΔABC

$\angle ACB = 90°$, $\angle ABC = 50°$

$\angle CAB = 180° - 140° = 40°$

12. (a); $\angle POQ = 2 \times 50° = 100°$

$\angle PTQ = 180° - 100° = 80°$

13. (c); AB = AC and AD = CD

(distance midpoint)

$AD^2 = AP \times AB$

$\left(\frac{AB}{2}\right)^2 = AP \times AB$

$AB^2 = 4AP \times AB$

$\Rightarrow AB = 4AP$

14. (a); In ΔAOM and BOM

OM = (common), OA = OB (radius)

$\angle AOM = \angle BOM$, $\Delta AOM \cong \Delta BOM$

AM = BM,

$\frac{AM}{BM} = \frac{1}{1}$

15. (a); TP = TR and TQ = TP

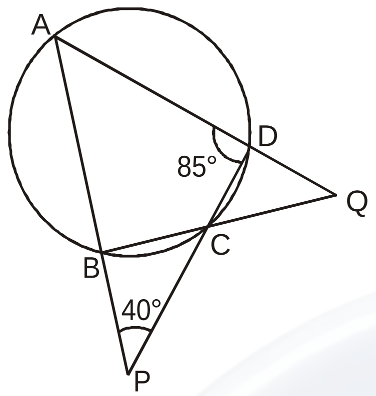
TQ = TP = TR

TQ : TR = 1 : 1

16. (b); $\angle AOC = 2 \times 60° = 120°$

$\angle ABC = \frac{1}{2} \times \angle AOC = \frac{1}{2} \times 120° = 60°$

17. (a);



$\angle ABC = 180° - 85° = 95°$

In ΔADP

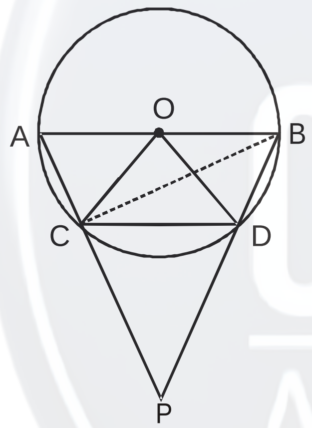
$\angle PAD = 180° - 85° - 40° = 180° - 125°$

$\angle PAD = 55°$

In ΔABQ

$\angle AQB = 180° - 55° - 95° = 30°$

18. (d);



OC = OD = CD = radius

$\angle COD = 60°$

$\angle CBD = \frac{1}{2} \times 60° = 30°$

$\angle ACB = 90°$ (angle is semicircle)

$\angle APB = 180° - \angle BCP - \angle CBD$

$= 180° - 90° - 30°$

$\angle APB = 60°$

19. (b); $\angle ACB = 90°$

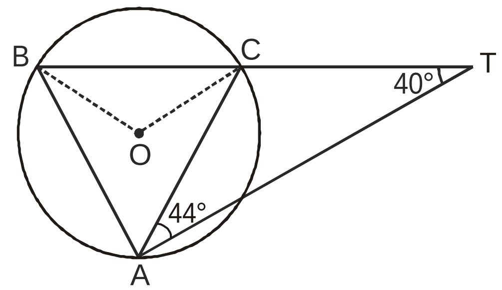
$\angle CBD = \frac{1}{2} \times 31 = 15.5$

$\angle APB = 180° - \angle PCB - \angle CBP$

$= 180° - 90° - 15.5°$

$\angle APB = 74.5°$

20. (d);



$\angle CAT = 44°$ (Given)

$\angle BTA = 40°$ (Given)

$\angle ACT = 180° - 44° - 40° = 96°$

$\angle CAT = \angle CBA = 44°$ (Alternate segment)

$\angle BCA = 180° - 96° = 84°$

$\angle BAC = 180° - 84° - 44 = 52°$

Angle subtended by BC at centre

$= 2 \times 52° = 104°$

21. (c); $\angle BOC = 2\angle BAC$

OB = OC

$\angle OBC = \angle OCB$

$\angle OBC = 180° - \angle BOC - \angle OCB$

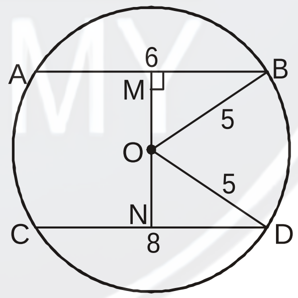
$= 180° - \angle BOC - \angle OBC$ $(\angle OBC = \angle OCB)$

$= 90° - \frac{\angle BOC}{2}$

$\angle BAC + \angle OBC = \angle BAC + 90° - \frac{\angle BOC}{2}$

$= \frac{\angle BOC}{2} + 90° - \frac{\angle BOC}{2} = 90°$

22. (b);



$BM = \frac{1}{2} AB$

BM = 3

$OM^2 = OB^2 - BM^2$

$OM^2 = 5^2 - 3^2 = 25 - 9 = 16$

OM = 4

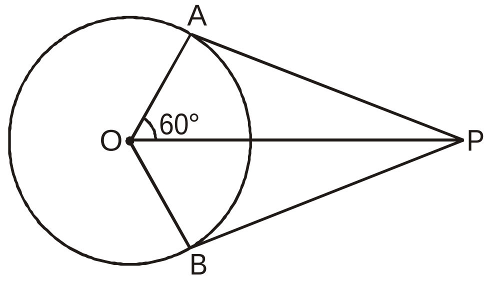
$ON^2 = OD^2 - DN^2$

$ON^2 = 5^2 - 4^2 = 25 - 16 = 9$

ON = 3

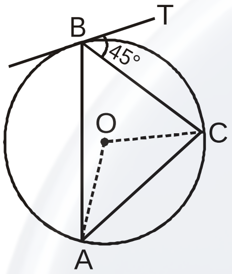
MN = OM + ON = 3 + 4 = 7 cm

23. (c);



$\angle AOP = 60°$ (Given)
$\angle OAP = 90°$ ($AP \perp OA$)
$\angle APO = 180° - 90° - 60°$
$\angle APO = 30°$
$\angle APB = 2\angle APO = 2 \times 30° = 60°$

24. (c);

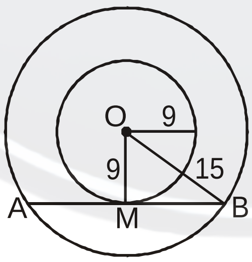


Let r = radius
CB = chord = 6 cm
$\angle CBT = 45°$ (Given)
$\angle CBT = \angle CAB$ (angle in a alternate segment)
$\angle COB = 2 \times 45° = 90°$
OC = OB = radius
In ΔCOB, $OC^2 + OB^2 = BC^2$
$2OC^2 = BC^2$

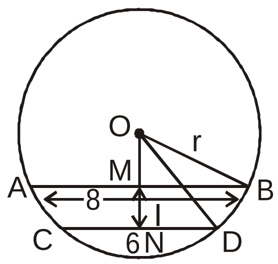
$$OC^2 = \frac{36}{2}$$

$r = OC = 3\sqrt{2}$ cm

25. (a); In ΔOBM
$OM^2 + BM^2 = OB^2$
$BM^2 = 15^2 - 9^2$
$BM^2 = 225 - 81$
$BM = \sqrt{144} = 12$ cm
$AB = 2BM$
$= 2 \times 12 = 24$ cm

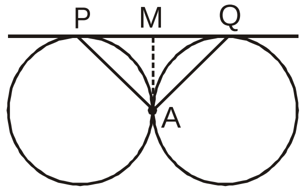


26. (a); Let OM = x
In ΔOMB
$r^2 = x^2 + 4^2$
In ΔOND
$r^2 = (x + 1)^2 + 3^2$
$x^2 + 4^2 = (x + 1)^2 + 3^2$
$x = 3$
$r^2 = 3^2 + 4^2 = 25$
$r = 5$ cm



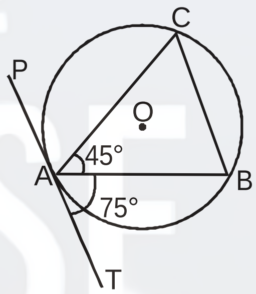
27. (d); Cannot be determined because value of Inradius and circumradius are side dependent unlike equilateral triangle.

28. (b);



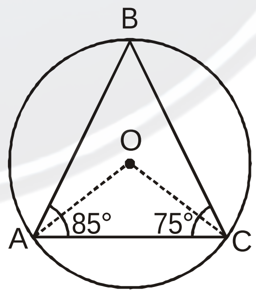
PM = AM
$\angle MPA = \angle MAP = x$
AM = MQ
$\angle MAQ = MQA = y$
In ΔAPQ
$\angle APQ + \angle AQP + \angle PAQ = 180°$
$x + y + \angle MAP + \angle MAQ = 180°$
$x + y + x + y = 180°$
$2(x + y) = 180°$
$x + y = 90°$
$\angle PAQ = 90°$

29. (c);



$\angle BAT = \angle ACB$
(angle in alternate segment)
$\angle ACB = 75°$
$\angle ABC = 180° - 45° - 75°$
$\angle ABC = 180° - 120° = 60°$

30. (c);



$\angle BAC = 85°$, $\angle BCA = 75°$
$\angle ABC = 180° - 85° - 75° = 180° - 160°$
$\angle ABC = 20°$, $\angle AOC = 40°$
$2\angle OAC = 180° - 40°$

$$\angle OAC = \frac{140°}{2} = 70°$$