Chapter - 10

PERMUTATION, COMBINATION PROBABILITY

Solutions

Foundation

1. **(b):** Required probability = $\frac{^6C_1 \times {}^2C_1}{^{52}C_2}$
$= \frac{6 \times 2}{\frac{52 \times 51}{1 \times 2}}$
$= \frac{6 \times 2}{26 \times 51}$
$= \frac{2}{221}$

2. **(d):** Required solution = $\frac{6}{15} \times \frac{6}{15} \times \frac{6}{15} + \frac{4}{15} \times \frac{4}{15} \times \frac{4}{15} + \frac{5}{15} \times \frac{5}{15} \times \frac{5}{15}$
$\Rightarrow \frac{216+64+125}{3375} = \frac{3}{25}$

3. **(c):** Number of ways such that four odd digits (5,5,3,3) can be arranged in 4 odd places
$= \frac{4!}{2! \times 2!} = 6\ ways$
Number of ways such that three even digits (6,4,4) can be arranged in 3 even places= $\frac{3!}{2!}$
=3 ways
Hence, the required number of ways=6 × 3 =18

4. **(a):** Total Possible outcome = 2^3 = 8
Possible outcome = 4 (HHH, THH, HHT, HTH)
Required Probability = $\frac{4}{8} = \frac{1}{2}$

5. **(b):** required probability =1-probability of three boys in first three positions
$=1 - {}^{10}C_3/{}^{13}C_3 = \frac{83}{143}$

6. **(a):** required probability = $\frac{7}{15} \times \frac{6}{14}$ = 0.20

7. **(a):** In the word CASTING, there are two vowels (A, I) and five consonants (C, S, T, N, G).
So, required probability = $\frac{6! \times 2!}{7!} = \frac{2}{7}$

8. **(d):** When two dices are rolled together,
Total number of possible cases= 36
Favourable cases= (1,1),(1,3),(1,5),(2,2),(2,4),(2,6),(3,1),(3,3),(3,5),(4,2),(4,4),(4,6),(5,1),(5,3),(5,5),(6,2),(6,4),(6,6)=18 cases
Required probability=$\frac{1}{2}$

9. **(a):** Total words than can be formed = 5! = 120
As vowels are together
i.e. H, R, T, [AE]
total words = 4! × 2! = 48
So, required probability = $\frac{48}{120} = \frac{2}{5}$

10. **(d):** Required number of the ways
$= {}^8C_3 \times {}^6C_4$
$= \frac{8 \times 7 \times 6 \times 5!}{3! \times 5!} \times \frac{6 \times 5 \times 4!}{2! \times 4!}$
= 840

11. **(d):** Possible outcomes = 9 [(1,4) (1,5)(2,3) (2,4)(3,2)(3,3)(4,1)(4,2)(5,1)]
So, required probability = $\frac{9}{36} = \frac{1}{4}$

12. **(d):** Possible outcomes = 12 [(1, 2) (1, 5) (2, 1) (2, 4) (3, 3) (3, 6) (4, 2) (4, 5) (5, 1) (5, 4) (6, 3) (6, 6)]
Required probability = $\frac{12}{36} = \frac{1}{3}$

13. **(a):** First prize can be given to any of 7 students, same way next prize can be given to any of 7 students and so on with every prize.
No. of ways = 7 X 7 X 7 X 7 X 7 = 7^5

14. **(b):** Required probability = $\frac{6}{14} = \frac{3}{7}$

15. **(d):** Total number of cases when two dices are rolled simultaneously=36
total cases of getting same number on both the dices=(1,1), (2,2), (3,3), (4,4), (5,5), (6,6) = 6
required probability=$1 - \frac{6}{36} = \frac{5}{6}$

16. **(d):** Required probability = $\frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$

17. **(d):** Probability of choosing a bag = $\frac{1}{2}$
Probability of choosing two red balls from Bag – A = $\frac{^7C_2}{^{15}C_2} = \frac{21}{105} = \frac{1}{5}$
Probability of choosing two red balls from Bag – B = $\frac{^xC_2}{^{x+7}C_2} = \frac{x\,(x-1)}{(x+6)(x+7)}$
ATQ,
$\frac{2}{15} = \frac{1}{2}\left[\frac{1}{5} + \frac{x\,(x-1)}{(x+6)\,(x+7)}\right]$
x = 3, –1
So, required numbers of balls is 3 as numbers of balls cannot be negative.

18. **(b):** Let yellow balls be x
P (at least a yellow ball) = $({}^xC_1 . {}^{15}C_1 + {}^xC_2)$ / ${}^{15+x}C_2$
$= \frac{(2\ X\ x\ X\ 15) + x\,(x-1)}{(15+x)(14+x)} = \frac{x^2 + 29x}{x^2 + 29x + 210} = \frac{17}{38}$
$21x^2 + 609x - 3570 = 0$
On solving, x = 5 (alternatively, solve equation using options)
No. of yellow balls = 5

19. **(c):** ATQ,
$\frac{^9C_2}{^{X+13}C_2} = \frac{4}{19}$
$\Rightarrow \frac{72}{(13+X)(12+X)} = \frac{4}{19}$
$\Rightarrow 342 = 156 + 25X + X^2$
$\Rightarrow X^2 + 25X - 186 = 0$
$\Rightarrow X^2 + 31X - 6X - 186 = 0 \ \Rightarrow X = 6$

20. (c): Probability of a Tiger = $\frac{7}{16}$

Let total Tiger → 7a

Total Animal → 16a

Now,

Total head = Total animal = 16a

So, Total legs ⇒ $\frac{16a}{2} \times 7 = 56a$

Now Ostrich → 2 legs

Tiger & Jackals → 4 legs

Total heads (Ostrich + Jackal) = 9a

Let no. of Ostrich → x

And no. of Jackal → 9a – x

ATQ

$x \times 2 + (9a - x) \times 4 = 56a - 7a \times 4$

$x = 4a$

So. no. of Jackals = 5a

Probability of choosing jackal = $\frac{5}{16}$

21. (c): ATQ,

$\frac{{}^{x}C_1 \times {}^{8}C_1}{{}^{x+15}C_2} = \frac{4}{15}$

$\frac{x \times 8}{\frac{(x+15)(x+14)}{2}} = \frac{4}{15}$

$\Rightarrow \frac{16x}{x^2 + 29x + 210} = \frac{4}{15}$

$\Rightarrow x^2 + 29x + 210 = 60x$

$\Rightarrow x^2 - 31x + 210 = 0$

$x^2 - 21x - 10x + 210 = 0$

$x(x - 21) - 10(x - 21) = 0$

$(x - 21)(x - 10) = 0$

x = 10, 21

Required answer = 10

22. (b): Let the number of red and black colored balls be x and y respectively.

ATQ,

$\frac{{}^{x}C_2}{{}^{(9+x)}C_2} = \frac{1}{7}$

$\Rightarrow \frac{x \times (x-1)}{(9+x) \times (8+x)} = \frac{1}{7}$

⇒ x = 6

Total number of balls=15

$\Rightarrow \frac{{}^{y}C_2}{{}^{15}C_2} = \frac{1}{7} \times \frac{100}{250}$

⇒ y=4

Number of yellow colored ball=15-(6+4) = 5

23. (a): required probability = $\frac{{}^{10}C_4 \times {}^{5}C_2}{{}^{15}C_6} = \frac{60}{143}$

24. (b): Letters which are to be used to make 5 letter words = E, C, U, O, U

No. of ways = $\frac{5!}{2!} = 60$

25. (a): Here, there is 5 green, 4 blue and 2 red balls

Probability of both ball being blue or green =

$\frac{{}^{5}C_2 + {}^{4}C_2}{{}^{11}C_2} = \frac{10+6}{55}$

$= \frac{16}{55}$

26. (c): total cards left in pack = 52 − 2 = 50

Required probability = $\frac{{}^{26}C_3}{{}^{50}C_3} = \frac{26 \times 25 \times 24}{50 \times 49 \times 48} = \frac{13}{98}$

27. (d): required players = 11

No. of ways = ${}^{15}C_{11} = 1365$

28. (b): Sample space when a dice is rolled twice = 36

Excluding the cases when number on second roll come equal or less than the first roll.

so, cases are

$[(1,1), (2,1), (2,2), (3,1), (3,2), (3,3)$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6,6)]$

⇒ [1 + 2 + 3 + 4 + 5 + 6] = 21

Favorable case = 36 – 21 = 15

Required probability = $\frac{15}{36} = \frac{5}{12}$

29. (d): Required arrangement = ${}^{7}C_4 \times {}^{6}C_2 \times 6!$

= 35 × 15 × 6!

= 525 × 6!

30. (c): The word father has 6 different letters

So, required no. of words = ${}^{6}P_4$

$= \frac{6!}{6!-4!} = \frac{6!}{2!}$

$= \frac{720}{2} = 360$ *words*

31. (b): Total outcome = 6 × 4 = 24

Feasible cases = $(6HH, 6HT, 6TH) = 3$

Required probability = $\frac{3}{24} = \frac{1}{8}$

= 0.125

32. (b): Possible cases = $4\{(3,6)(4,5)(5,4)(6,3)\}$

Required probability = $\frac{4}{36} = \frac{1}{9}$

33. (a): Required probability = $\frac{{}^{4}C_1 \times {}^{48}C_1}{{}^{52}C_2} + \frac{{}^{4}C_2}{{}^{52}C_2}$

$= \frac{32}{221} + \frac{1}{221} = \frac{33}{221}$

34. (a): Required Probability = $\frac{5}{23} + \frac{8}{23} = \frac{13}{23}$

Moderate

1. (d): The word 'RAINBOW' contains three vowels (A, I, O) and rest are consonant.

Total arrangement=${}^{3}C_1$(_ 5! _ _ _) ${}^{5}C_1$ = 3 × 120 × 5 = 1800

2. (b): Total balls=25 balls

Total prime number between 1 to 25 =9

Total perfect square=5

Probability of selecting 1st ball=$\frac{9}{25}$

Probability of selecting 2nd ball=$\frac{1}{5}$

Required probability=$\frac{9}{25} \times \frac{1}{5} = \frac{9}{125}$

3. (b): Let total female = x
So, total male = (12 – x)
$\frac{(12-x)(11-x)}{12\times11} - \frac{x(x-1)}{12\times11} = \frac{1}{6}$
$132 - 23x + x^2 - x^2 + x = 22$
22x = 110
x = 5
∴ Female=5
So male = (12 – 5) = 7
Required difference = 7 – 5 = 2

4. (b): The probability of selecting one bag out of the two bags is $= \frac{1}{2}$
So required probability $= \frac{1}{2}\left(\frac{{}^6C_1\times{}^4C_1}{{}^{10}C_2} + \frac{{}^4C_1\times{}^4C_1}{{}^8C_2}\right) = \frac{1}{2}\left(\frac{8}{15}+\frac{4}{7}\right) = \frac{58}{105}$

5. (d): The various possibilities are described as
i) Number starts with 3
no. of ways = 1×5^3
= 125 (*since other digits can be repeated*)
ii) Number doesn't start with 3
no. of ways = $4 \times 5^2 \times 3 = 300$ (digit 3 can appear at 3 different places i.e. in 2nd ,3rd and 4th places in the number and 0 can't be placed as 1st digit of the number)
total ways = 125 + 300 = 425

6. (a): the probability of solving the problem by
Jindal = 3/5
Avi = $1 - \frac{1}{4} = \frac{3}{4}$
The probability of not solving the problem by
Jindal = $1 - \frac{3}{5} = \frac{2}{5}$
Avi = 1/4
Required probability=$\frac{3}{5}\times\frac{1}{4}+\frac{2}{5}\times\frac{3}{4}+\frac{3}{5}\times\frac{1}{4} = \frac{3+6+3}{20} = \frac{3}{5}$

7. (b): seat pattern is 2X2 which means 40 seats are distributed in 10 rows
So there are only 20 window seats and 20 non window seats
10 window seats are already occupied, remaining window seats = 20 – 10 = 10
Ways to select window seat = ${}^{10}C_2$
Shreyas wants seats together, so non window seats should be next to selected window seats.
No. of ways for non-window seats = ${}^{10}C_2$
Total no of ways = ${}^{10}C_2 \times {}^{10}C_2$
$= \frac{10\times9}{2}\times\frac{10\times9}{2} = 2025$

8. (d): Number of green balls=16-9=7
Number of red balls=7-4=3
Number of blue balls=9-3=6
ATQ,
Required probability=$\frac{{}^3C_1\times{}^6C_1\times{}^7C_1}{{}^{16}C_3} = \frac{9}{40}$

Solution (9 – 10):
Let red balls & green balls in the bag are 4x & 5x respectively
ATQ –
$\frac{4x+3}{5x-1} = \frac{11}{9}$
36x + 27 = 55x –11
19x = 38
x = 2
Number of red balls = 8
Number of green balls = 10

9. (d): ATQ –
$\frac{n}{18+n} + \frac{8}{18+n} = \frac{2}{3}$
36 + 2n = 3n + 24
n = 12

10. (d): In order to get maximum red balls left in bag, the only condition which satisfies is that all balls taken out from the bag are of green color.
So. Total balls left in bag = 10 – 5 + 8 = 13
Required probability = $\frac{8}{13}$

11. (c): Let total number of red balls = x
So, total number of blue balls = (12 – x)
ATQ –
$\frac{x(12-x)}{6\times11} = \frac{35}{66}$
$12x - x^2 = 35$
$x^2 - 12x + 35 = 0$
x(x – 5) – 7 (x – 5) = 0
x = 5 & 7
Now new number of blue balls in bag = (7 + n)
Given, $\frac{(5+n)}{(10+n)} = \frac{9}{14}$
70 + 14n = 90 + 9n
5n = 20
n = 4

12. (a): ∴ P (two different colored balls) = P (1st blue and 2nd green) + P (1st blue and 2nd black) + P (1st blue and 2nd white) + P (1st black and 2nd green) + P (1st black and 2nd white)
$= \left(\frac{4}{9}\times\frac{1}{5}\right)+\left(\frac{4}{9}\times\frac{3}{10}\right)+\left(\frac{4}{9}\times\frac{1}{2}\right)+\left(\frac{5}{9}\times\frac{1}{5}\right)+\left(\frac{5}{9}\times\frac{1}{2}\right) = \frac{5}{6}$

13. (b): red shirts = 15 – 7 = 8
Required probability = $\frac{{}^8C_2}{{}^{15}C_2} = \frac{8\times7}{15\times14} = \frac{4}{15}$

14. (d): possible cases
(1boy, 3 girls) = ${}^4C_1 \times {}^5C_3 = 40$
(2 boys, 2 girls) = ${}^4C_2 \times {}^5C_2 = 60$
Total ways = 40 + 60 = 100

15. (b): ATQ,
$\frac{{}^xC_1\times{}^5C_1}{{}^{x+11}C_2} = \frac{9}{38}$
$\frac{x\times5}{(x+11)(x+10)}\times2 = \frac{9}{38}$
$\Rightarrow \frac{10x}{x^2+21x+110} = \frac{9}{38}$
$\Rightarrow 380x = 9x^2 + 189x + 990$
$\Rightarrow 9x^2 - 191x + 990 = 0$
$9x^2 - 110x - 81x + 990 = 0$
x (9x – 110) –9(9x – 110) = 0
(9x – 110) (x – 9) = 0
x = 9, $\frac{110}{9}$
So, x = 9

16. (b): According to question
$\frac{2000\times12}{(2000+x)8} = \frac{5}{4}$
50 × 48 = 2000 + x
x = Rs 400

17. (a): Required probability $= \frac{^{26}C_2 \times {}^{26}C_1}{^{52}C_3}$

$= \frac{8450}{22100} = \frac{13}{34}$

18. (b): In 'COMBINATION', there are 5 vowels (O, O, I, I, A)

So, required number of words $= \frac{7! \times 5!}{2! \times 2! \times 2!}$

$= \frac{604800}{8} = 75600$

19. (b): Probability of choosing a bag $= \frac{1}{2}$

Required probability $= \frac{1}{2}\left[\frac{^4C_2}{^{12}C_2} + \frac{^9C_2}{^{15}C_2}\right]$

$= \frac{1}{2}\left[\frac{6}{66} + \frac{36}{105}\right]$

$= \frac{1}{2}\left[\frac{1}{11} + \frac{12}{35}\right]$

$= \frac{1}{2} \times \left[\frac{35+132}{385}\right] = \frac{167}{770}$

20. (b): Let total number of girls in classroom = x

ATQ—

$\frac{^xC_1}{^{32}C_1} = \frac{3}{8}$

$\frac{x}{32} = \frac{3}{8}$

x = 12

Total number of boys in class = 32 – 12 = 20

Required probability $= \frac{12 \times 20 \times 2}{31 \times 32} = \frac{15}{31}$

21. (c): Required probability

$= \frac{2}{5} \times \frac{1}{6} \times \frac{3}{7} + \frac{3}{5} \times \frac{5}{6} \times \frac{3}{7} + \frac{3}{5} \times \frac{1}{6} \times \frac{4}{7} + \frac{2}{5} \times \frac{5}{6} \times \frac{3}{7} + \frac{2}{5} \times \frac{1}{6} \times \frac{4}{7} + \frac{3}{5} \times \frac{5}{6} \times \frac{4}{7} + \frac{2}{5} \times \frac{5}{6} \times \frac{4}{7}$

$= \frac{201}{210} = \frac{67}{70}$

Or

Probability that no one is selected $= \frac{3}{5} \times \frac{1}{6} \times \frac{3}{7}$

$= \frac{3}{70}$

Required probability $= 1 - \frac{3}{70} = \frac{67}{70}$

22. (c): Let the number of green, red and white ball in the bag be 3x, 4x and 5x respectively.

ATQ

$\frac{4xc_1 \times 5xc_1}{12xc_2} = \frac{2}{7}$

$x = 3$

Total number of balls=36

Required probability$= \frac{12c_2}{36c_2} = \frac{11}{105}$

23. (c): Even number card = 2, 4, 6, 8, 10

Total even number card = 4 × 5 = 20

Favorable cases = $^{20}C_2$

Total cases = $^{52}C_2$

Probability $= \frac{^{20}C_2}{^{52}C_2} = \frac{20 \times 19}{52 \times 51} = \frac{95}{663}$

24. (c): Required probability $= \frac{^5C_3}{^{16}C_3} + \frac{^7C_3}{^{16}C_3} + \frac{^4C_3}{^{16}C_3}$

$= \frac{10+35+4}{560} = \frac{7}{80}$

25. (c): Starting four prime number = 2, 3, 5, 7

Total two-digit numbers can be formed = 4 × 4 = 16

Numbers which are divisible by 3

= {27, 72, 57, 75, 33}

Required probability $= \frac{5}{16}$

26. (d): Ways to select 3 balls out of 8 balls = 8_{C_3}

Ways to select one red ball = 2_{C_1}

Ways to select two black ball = 3_{C_1}

Ways to select one white balls = 3_{C_1}

∴ Required probability

$= \frac{2_{C_1} \times 3_{C_1} \times 3_{C_1}}{8_{C_3}} = \frac{9}{28}$

27. (c): Required Probability $= \frac{^5C_3 + {}^7C_3 + {}^4C_3}{^{16}C_3}$

$= \frac{10+35+4}{560} = \frac{7}{80}$

28. (d): Four digits number with all four digits odd = 5 × 5 × 5 × 5 = 625

29. (d): To put 5 different chocolate in identical boxes →

Boxes treated as one box → 1 way

Choices vary in selection of chocolates

⇒ to choose 5 chocolates → 5!

<u>Answer</u>

_______________5! × 1 ⇒ 120

30. (d): Lets total number apple in basket = x

Atq,

$\frac{^xC_1}{^{36}C_1} = \frac{1}{6}$

$\frac{x}{36} = \frac{1}{6}$

x = 6

Total number of banana in basket = 36 – 6 = 30

31. (c):

Anurag	*Anil*	*Anand*
1	3	3
3	1	3
3	3	1

One particular piece out of seven can be selected in 7 ways & to second person it can be given in $^6C_3$

Required number of ways = 3 × $^7C_1$ × $^6C_3$ = 7×20×3 = 420

32. (d): If we pick exactly 4 guavas, 6 mangoes and 7 pears from the basket, the total number of fruits picked from the basket will be 17.

If one more fruit is picked from the basket, irrespective of what it is, it can be said that at least 5 guavas or at least 7 mangoes or at least 8 pears have been picked from the basket.

Hence, required number of fruits=17+1=18

33. (b): Probability of not getting a blue ball in two consecutive trials$= \frac{9}{25} = \frac{81}{225}$

i.e. number of blue balls=15-9=6

probability of getting two green balls in two consecutive trials$= \frac{1}{25} = \frac{9}{225}$

i.e. number of green balls=3

so, the number of red balls=6

required probability=$\frac{6}{15} \times \frac{3}{15} \times \frac{6}{15} \times 3!$

$= \frac{24}{125}$

34. (d): If we pick exactly 4 guavas, 5 mangoes, 6 oranges and 7 pears then total number of fruits picked from the basket will be 22.

And when we further pick 1 more fruit irrespective of what it is, we have at least 5 guavas or at least 6 mangoes or at least 7 oranges or at least 8 pears.

So the required minimum, number of fruits picked from the basket= 22+1=23

35. (d): Probability of getting mango = $\frac{1}{2} \times \frac{^3C_1}{^9C_1} + \frac{1}{2} \times \frac{^7C_1}{^9C_1}$

$= \frac{1}{2} \times \frac{3}{9} + \frac{1}{2} \times \frac{7}{9} = \frac{5}{9}$

Probability of not getting mango = $1 - \frac{5}{9} = \frac{4}{9}$

Permutation And Combination Formulas

Factorial of a natural number n.
n! = 1 × 2 × 3 × 4 × ....× n

Permutation Formula for Bank Exams:
nPr = (n!) / (n-r)!

Combination Formula for Bank Exams:
nCr = (n!) / r!(n-r)!

The relationship between permutation and combination for r things taken from n things
nPr = r! × nCr

Difference Between the Permutation and Combination

Permutation	Combination
Permutations are used if order or sequence of arrangement is needed	Combinations are used when only the number of possible groups are to be found, and the order or sequence of arrangements is not needed
Permutations are used for things of a different kind	Combinations are used for things of a similar kind
The permutation of two things from three given things x, y, z is xy, yx, yz, zy, xz, zx	The combination of two things from three given things x, y, z is xy, yz, zx
For different possible arrangements of things nPr=n!/(n-r)!	For different possible selection of things nCr =n!/r!(n-r)!