Chapter - 1

Number System and Simplification

Number System Tips and Tricks

Number System	Examples
Whole Number	0, 1, 2, 3, 4, 5...
Natural Number	1, 2, 3, 4, 5, 6...
Integers	...-3, -2, -1, 0, 1, 2, 3, 4, 5,...
Prime Number	3, 5, 7, 11, 13, 17.....
Co-Prime Number	HCF = 1
Composite Number	4, 6, 8, 9, 12, 14, 15.....
Even Number	2, 4, 6, 7, 8, 10...
Odd Number	1, 3, 5, 7, 9....
Rational Number	In 'p/q' form wherein p & q are integers and q is not equal to 0

Foundation

Solutions

1. (b); The prime numbers Less than 31 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

$\therefore$ required sum = 2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 + 23 + 29 = 129

2. (b); Total even numbers from 1 to 50 = 25

Total even numbers from 1 to 20 = 10

Sum of even numbers = n(n + 1)

Required sum = sum of even numbers from 1 to 50 – sum of even numbers from 1 to 20

= 25(25 + 1) – 10(10 + 1)

= 25 × 26 – 10 × 11 = 540

3. (c); sum of first n natural numbers $=\frac{n(n+1)}{2}$

$\therefore$ sum of 1^{st} 25 natural numbers

$=\frac{25\times(25+1)}{2}=25\times13$

i.e. 13 is one of the factor

4. (a); $(4)^{1793/4}\times5\times1$

4 × 5 × 1 = 20 So, unit digit is 0.

5. (a); 1 + 1 – 6 + 5 = 1

6. (c); If square of any prime number is divided by 24 then remainder is always 1.

so, $\frac{(1+1+1+1)}{24}=\frac{4}{24}$ i.e 4 is unit digit.

7. (c); (1 + 3 + 5 + ... + 97) – (2 + 4 + 6 + ... + 98)

$n_1=\frac{97+1}{2}=49$, $n_2=\frac{98}{2}=49$

sum = $n_1^2 - n_2(n_2+1) = 49^2 - 49\times50 = -49$

8. (a); $\frac{250}{2}=125$, $\frac{125}{5}=25$, $\frac{25}{5}=5$, $\frac{5}{5}=1$

i.e. required numbers of zero = 25 + 5 + 1 = 31

9. (b); 24

10. (c); 25

11. (d); Required remainder = $\frac{(21+28)}{33}=16$

12. (d); Let quotient = x

divisor = 7x also divisor = 3 × (remainder)

= 3 × 28 = 84

7x = 84, x = 12

Dividend = Divisor × Quotient + Remainder

= 84 × 12 + 28 = 1036

13. (c); Since it is form of $\frac{a^n}{a+1}$

i.e. $\frac{17^{200}}{17+1}$

∴ Remainder = 1, Since n is even positive integer

14. (d); A number is exactly divisible by 18 if it is divisible by 2 and 9 both.

since, 65043 is not divisible by 2, so it is not divisible by 18.

15. (a); by checking option

$2^{96} + 1 = (2^{32})^3 + 1^3 = (2^{32} + 1)(2^{64} - 2^{32} + 1)$

16. (b); Decimal equivalent of fractions

$\frac{4}{9}=0.44;\ \sqrt{\frac{9}{49}}=\frac{3}{7}=0.43$

$(0.8)^2 = 0.64$

∴ Least number = 0.43 = $\sqrt{\frac{9}{49}}$

17. (c); Expression = 0.121212 ...

$=0.\overline{12}=\frac{12}{99}=\frac{4}{33}$

[Since, 12 is repeating after decimal]

18. (b); Decimal equivalent of fractions

$\frac{15}{16}=0.94,\ \frac{19}{20}=0.95,\ \frac{24}{25}=0.96,\ \frac{34}{35}=0.97$

∴ Least fraction = $\frac{15}{16}$

19. (b); Given, $1^3 + 2^3 + \ldots 9^3 = 2025$

Then, $(0.11)^3 + (0.22)^3 + \ldots + (0.99)^3$

$=\left(\frac{11}{100}\right)^3+\left(\frac{22}{100}\right)^3+\ldots+\left(\frac{99}{100}\right)^3$

$=\left(\frac{11}{100}\right)^3\left(1^3+2^3+\ldots+9^3\right)$

$=\frac{1331}{1000000}\times 2025$

[$\because\ 1^3 + 2^3 + \ldots + 9^3 = 2025$]

$=\frac{2695275}{1000000}=2.695275\approx 2.695$

20. (b); Decimal equivalent of fractions

$0.9=\frac{9}{10},\ 0.\overline{9}=\frac{9}{9}=1,\ 0.0\overline{9}=\frac{9}{90}=\frac{1}{10}$

and $0.\overline{09}=\frac{9}{99}=\frac{1}{11}$

∴ $0.\overline{9}$ is greatest.

21. (b); Natural numbers between 3 and 200

= 200 – 3 = 197

Now divide 197 by 7

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   28
7)197
  14
  ---
   57
   56
   --
    1
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So 28 natural numbers are there

22. (d); Let the consecutive odd no. are x, x + 2, x + 4

x + x + 2 + x + 4 = 87

3x + 6 = 87

$x=\frac{81}{3}=27$

so, smallest number is 27.

23. (b); 7^{105}

Cyclicity of 7 is 4.

So $\frac{105}{4}$ = Remainder is 1.

7^1 = Unit digit

24. (c); $5^{71} + 5^{72} + 5^{73}$

$5^{71}(1 + 5 + 5^2)$

$5^{71} \times 31$

$5^{70} \times 155$

so 155 divides the expression completly

25. (a); We know that $2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16$

Remainder $=\frac{33}{4}=1$.

Unit's digit in 2^{33} = unit digit in 2^1

Hence units digit = 2

Remainder on division by 10 = 2.

26. (b); Remainder = 16
Divisor = 24
Let number = x
x = 24y + 16 where y is quotient.
Since 24 is a multiple of 12

Remainder = $\frac{16}{12} = 4$

27. (c); $\frac{3^{21}}{5}$

$\frac{(3^4)^5 \times 3}{5} = \frac{(81)^5 \times 3}{5}$

$= \frac{1^5 \times 3}{5}$

so, remainder = 3

28. (d); Let the two digit number be xy
xy xy = xy × 100 + xy
= xy (100 + 1) = 101 xy

29. (d); Numbers which are multiple of both 10, 13 will be multiple of 130 also
Numbers less then 1000 which are multiple of both 10 and 13

$= \frac{1000}{130} = 7$

30. (c); $\frac{x}{\sqrt{0.25}} = 25$
x = 25 × (0.5) = 12.5

31. (c); Required number = (Sum of digits at odd places) – Sum of digits at even place)
= (2 + 6 + 0) – (8 + 3 + 4) = – 7
smallest number to be added = 7

32. (d); Factor of 1008
= 2 × 2 × 2× 2 × 3 × 3 × 7
so number is divided by 7 to make it perfect square.

33. (b); $1^2 + 2^3 + 3^2 ...... + 10^2$

$= \frac{n(n+1)(2n+1)}{6} = \frac{10 \times 11 \times 21}{6} = 385$

34. (b); Sum of squares from 1 to 20 – Sum of squares from 1 to 9

$= \frac{20 \times 21 \times 41}{6} - \frac{9 \times 10 \times 19}{6} = 2870 - 285 = 2585$

35. (c); Let four consecutive natural numbers are
1, 2, 3, 4
1 × 2 × 3 × 4 = 24
So 24 is a natural number which divides four consecutive natural number completely

36. (d); Given, $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = 441$
$2^3 + 4^3 + 6^3 + 8^3 + 10^3 + 12^3$
$= 2^3(1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3)$
$= 2^3 \times 441 = 3528$

37. (a); $\frac{2}{3}, \frac{5}{6}, \frac{11}{15}$ and $\frac{7}{8}$
Using cross multiplication method.

$\frac{2}{3} \times \frac{5}{6} = 12 < 15$

So, $\frac{5}{6} > \frac{2}{3}$

$\frac{5}{6} \times \frac{11}{15} = 75 > 66$

So , $\frac{5}{6}$ is greater than $\frac{11}{15}$

$\frac{5}{6} \times \frac{7}{8} = 40 < 42$

So $\frac{7}{8}$ is the greatest fraction.

38. (b); $0.4\overline{23} = \frac{423-4}{990} = \frac{419}{990}$

39. (b); 0.393939 ....

$= 0.\overline{39} = \frac{39}{99} = \frac{13}{33}$

40. (d); Let number = y.
According to question

$\frac{1}{3} \times \frac{1}{4} y = 15$, y = 180

so, $\frac{3}{10} y = \frac{3}{10} \times 180 = 54$

Moderate

1. (c); $\frac{2}{3} \times \frac{3}{5}$: 10 > 9. Taking greater of these two fractions and the next one. $\frac{2}{3} \times \frac{8}{11}$: 22 < 24. Taking greater of these two fractions and the next one. $\frac{8}{11} \times \frac{7}{9}$: 72 < 77. Taking greater of these two fractions and the next one. $\frac{7}{9} \times \frac{11}{17}$: 119 > 99. $\frac{7}{9}$ is the largest

2. (d); $\frac{7}{12} \times \frac{13}{24}$: 168 > 156. $\frac{13}{24} \times \frac{9}{17}$: 221 > 216

$\frac{7}{12} > \frac{13}{24}$ and $\frac{13}{24} > \frac{9}{17}$

Hence descending order = $\frac{7}{12} > \frac{13}{24} > \frac{9}{17}$

3. (a); $1\frac{1}{2} + 11\frac{1}{2} + 111\frac{1}{2} + 1111\frac{1}{2} + 11111\frac{1}{2}$

= [1 + 11 + 111 + 1111 + 11111] + $\left[\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right]$

$= 12345 + 2\frac{1}{2} = 12347\frac{1}{2}$

4. (c); $3\frac{1}{3} + 33\frac{1}{3} + 333\frac{1}{3} + 3333\frac{1}{3} + 33333\frac{1}{3}$

= [3 + 33 + 333 + 3333 + 33333] + $\left[\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right]$

$= 37035 + 1\frac{2}{3} = 37036\frac{2}{3}$

5. (b); $\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72}$

$= \frac{1}{2\times3} + \frac{1}{3\times4} + ... + \frac{1}{8\times9}$

$= \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + ... + \frac{1}{8} - \frac{1}{9}$

$= \frac{1}{1}\left[\frac{1}{2} - \frac{1}{9}\right] = \frac{7}{18}$

6. (b); $\frac{1}{5.9} + \frac{1}{9.13} + \frac{1}{13.17} + ... \frac{1}{61.65} = ?$

Using formula:

$$\frac{+1}{\text{Difference of denominator value}}\left[\frac{1}{\text{First value}} - \frac{1}{\text{Last value}}\right]$$

$$= \frac{1}{4}\left[\frac{1}{5} - \frac{1}{65}\right] = \frac{1}{4}\left[\frac{13-1}{65}\right] = \frac{1}{4}\left[\frac{12}{65}\right] = \frac{3}{65}$$

7. (c);

$$x = \cfrac{3}{2 + \cfrac{2}{2 + \cfrac{2}{2 + \frac{2}{3}}}} \quad 2+\frac{2}{3} = \frac{8}{3}$$

$$= \cfrac{3}{2 + \cfrac{2}{2 + \cfrac{2}{\frac{8}{3}}}} \quad = 2 + \frac{2}{1} \times \frac{3}{8} = 2 + \frac{3}{4} = \frac{11}{4}$$

$$= \cfrac{3}{2 + \cfrac{2}{\frac{11}{4}}} = \cfrac{3}{2 + \frac{2}{1} \times \frac{4}{11}}$$

$$= \cfrac{3}{2 + \frac{2}{1} \times \frac{4}{11}} \quad = 2 + \frac{8}{11} = \frac{30}{11}$$

$$= \cfrac{3}{\frac{30}{11}} = \frac{3}{1} \times \frac{11}{30} = \frac{11}{10} = 1.1$$

8. (a);

$$x + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4 + \frac{1}{5}}}} = 12$$

$$12 = x + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{\frac{21}{5}}}} = x + \cfrac{1}{2 + \cfrac{1}{3 + \frac{5}{21}}}$$

$$12 = x + \frac{1}{2 + \frac{1}{\frac{68}{21}}} = x + \frac{1}{\frac{157}{68}} = x + \frac{68}{157}$$

$$x = 12 - \frac{68}{157}$$

$157x = 1884 - 68 = 1816$

$$x = \frac{1816}{157}$$

9. (a); $5.\overline{12} + 3.\overline{21} + 4.\overline{31} = 5\frac{12}{99} + 3\frac{21}{99} + 4\frac{31}{99}$

$$= (5+3+4) + \frac{64}{99} = 12\frac{64}{99}$$

10. (d); $5.\overline{76} - 2.\overline{3} = 5\frac{76}{99} - 2\frac{3}{9} = 3\frac{43}{99} = 3.\overline{43}$

11. (b); Given, $-1 \le \frac{2x-7}{5} \le 1$

$\Rightarrow -5 \le 2x - 7 \le 5$ ... (i)

$\Rightarrow -5 + 7 \le 2x - 7 + 7 \le 5 + 7$

[by adding 7 in eq. (i)]

$\Rightarrow 2 \le 2x \le 12$

$\Rightarrow 1 \le x \le 6$

So, number of values of x = 3 (2, 3 and 5)

12. (c); Let the required number be x.

According to the question,

$x^2 + x = 2 \times 3 \times 5$

$\Rightarrow x^2 + x - 30 = 0$

$\Rightarrow x^2 + 6x - 5x - 30 = 0$

$\Rightarrow x(x + 6) - 5(x + 6) = 0$

$\Rightarrow (x - 5)(x + 6) = 0$

$\therefore x = 5$

13. (b); Required number between $\frac{1}{2}$ and $\frac{3}{5}$

$$\Rightarrow \frac{\frac{1}{2} + \frac{3}{5}}{2}$$

$$\Rightarrow \frac{5+6}{20} = \frac{11}{20} \approx \frac{4}{7}$$

14. (c); Let two consecutive even numbers are x and (x+2).

$\therefore$ According to the question,

$(x+2)^2 - x^2 = 84$

$\Rightarrow x^2 + 4x + 4 - x^2 = 84$

$\Rightarrow 4x = 84 - 4 = 80$

$\Rightarrow x = \frac{80}{4} = 20$

Two numbers are 20 and 22.

$\therefore$ The required sum = 20 + 22 = 42

15. (d); Required sum = (sum of natural numbers from 1 to 100) – (sum of natural numbers from 1 to 49.)

Sum of [1 + 2 + 3 + 4 + ... + 100]

$$= \frac{n(n+1)}{2} = \frac{100(101)}{2} = 5050$$

and sum of [1 + 2 + 3 + 4 ..... + 49]

$$= \frac{n(n+1)}{2} = \frac{50(49)}{2} = 1045$$

Hence, sum of [50 + 51 + 52 + 53 + ... + 100]

= 5050 – 1045 = 4005

16. (b); Given, $(1001)^{2008} + 1002$

Unit digit of $(1001)^{2008} = 1$

Last digit of 1002 = 2

$\therefore$ The last digit = 1 + 2 = 3

17. (d); Given, $7^{71} \times 6^{63} \times 3^{65}$

Then, $7^1 = 7$, $7^2 = 49$, $7^3 = 343$, $7^4 = 2401$

$\therefore$ Unit digit of $(7)^{71} = 3$

$3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$

Unit digit of $(3)^{65} = 3$

Unit digit of $(6)^{63} = 6$

$\therefore$ Product = 3 × 6 × 3 = 54

$\therefore$ Unit digit = 4

18. (c); $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$

Unit digit repeats itself after 4 powers.

Remainder of $\frac{23}{4} = 3$

$\therefore (22)^{23} = (22)^3 = 2^3 = 8$

Unit digit = 8.

19. (c); Given, $(2153)^{167}$

Then, remainder of $\frac{167}{4} = 3$

$\therefore$ Unit digit of 3^3 (i.e., 27) = 7

20. (b); Such number is always divisible by 9. To make it clear, you can take some example.

Example:

496 – (4 + 9 + 6) = 477,

which is divisible by 9.

971 – (9 + 7 + 1) = 954,

which is divisible by 9.

21. (d); According to the question,
Divisor = 5 × Remainder
= 5 × 46 = 230

Quotient = $\frac{230}{10} = 23$

Dividend = Divisor × Quotient + Remainder
Dividend = 230 × 23 + 46 = 5290 + 49 = 5336
∴ Dividend = 5336

22. (c); Given, a and b are odd positive integers.
$a^4 - b^4 = (a^2 + b^2)(a + b)(a - b)$
Let two positive odd integers be 1 and 3.
∴ Required number
$= (1^2 + 3^2)(3 + 1)(3 - 1) = 80$
Which is divisible by 8.

23. (d); Required remainder = $\frac{\text{Last remainder}}{\text{New divisor}}$

Required remainder = $\frac{36}{17} = 2\frac{2}{17}$

∴ Remainder = 2

24. (c); Remainder = $\frac{\text{Last remainder}}{\text{New divisor}}$

Remainder = $\frac{63}{29} = 2\frac{5}{29} = 5$

25. (a); Decimal equivalent of fractions

$\frac{2}{3} = 0.67$; $\frac{5}{6} = 0.83$

$\frac{11}{15} = 0.73$; $\frac{7}{8} = 0.875$

∴ Greatest fractions is 7/8.

26. (d); $67^{67} = (68 - 1)^{67}$ when divided by 68, leaves remainder $(-1)^{67} = -1$
∴ Required remainder = – 1 + 67 = 66

27. (d); We know that,
Sum of squares of 1st n natural numbers

$= \frac{n(n+1)(2n+1)}{6}$

Required sum = (Sum of squares of natural numbers from 1 to 10) – 1^2

$= \frac{10(10+1)(2\times10+1)}{6} - 1^2 = \frac{10\times11\times21}{6} - 1$

= 385 – 1 = 384

28. (b); Here, $1^3 + 2^3 + ... + 10^3 = 3025$
Now, 4 + 32 + 108 + ... + 40000
= 4(1 + 8 + 27 + ... + 1000)
$= 4(1^3 + 2^3 + 3^3 + ... + 10^3)$
= 4 × 3025 = 12100

29. (d); To find the smallest fraction first we have to find the decimal equivalent of fractions

$\frac{7}{6} = 1.166$, $\frac{7}{9} = 0.777$, $\frac{4}{5} = 0.8$ and $\frac{5}{7} = 0.714$

Therefore, the smallest number is $\frac{5}{7}$.

30. (b); $0.\overline{001} = \frac{1}{999}$