Chapter - 5 Laws Of Motion

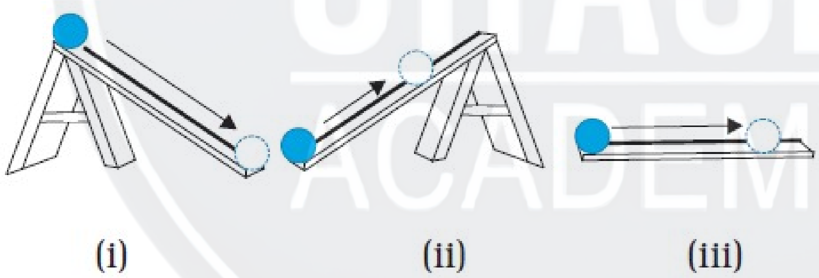
ARISTOTLE'S FALLACY

- Aristotelian law of motion can be stated as : **An external force is required to keep a body in motion**
- Galileo disproved this by experiments using inclined planes.

GALILEO'S EXPERIMENTS

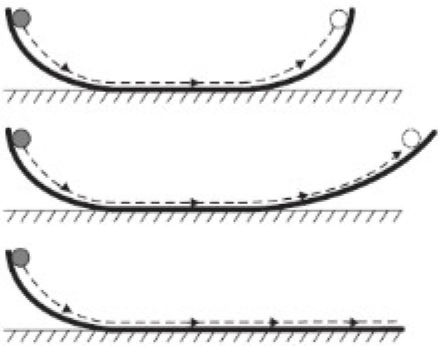
Using single inclined plane

- Galileo studied motion of objects on an inclined plane.
- Objects
    1. moving down an inclined plane accelerate
    2. objects moving up retard
    3. Motion on a horizontal plane is an intermediate situation.



- Galileo concluded that an object moving on a frictionless horizontal plane should move with constant velocity.

Using Double inclined plane



- A ball released from rest on one of the planes rolls down and climbs up the other.
- When friction is absent, the final height of the ball is the same as its initial height.
- If the slope of the second plane is decreased and the experiment repeated, the ball will still reach the same height, but it will travel a longer distance.
- When the slope of the second plane is zero (i.e., is a horizontal) the ball travels an infinite distance.
- If there were no friction, the ball would continue to move with a constant velocity on the horizontal plane.
- Thus, according to Galileo, the state of rest and the state of uniform linear motion (motion with constant velocity) are equivalent.
- If the net external force is zero, a body at rest continues to remain at rest and a body in motion continues to move with a uniform velocity.

NEWTON'S LAWS OF MOTION

- Starting from Galileo's ideas, Newton formed three laws of motion.

NEWTON'S FIRST LAW OF MOTION – THE LAW OF INERTIA

- **Everybody continues to be in its state of rest or of uniform motion in a straight line unless compelled by some external**

force to act otherwise.

INERTIA

- The inability of a body to change its state of rest or uniform motion along a straight line.
- Inertia means resistance to change.

Inertia of rest (static inertia)

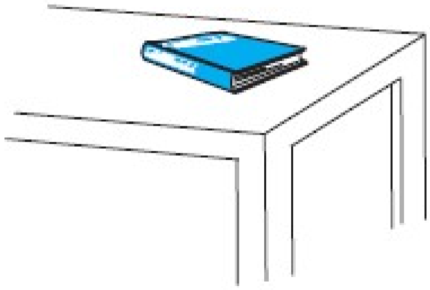
- The property of an object by virtue of which it remains in its state of rest.
- Example:
    1. a person standing in a bus tends to fall backward when the bus starts suddenly
    2. Fruits fall down due to inertia of rest when the branches of a tree are shaken
    3. Dust particles on a carpet fall if we beat the carpet with a stick
    4. With a quick pull, a table cloth can be removed from a dining table without disturbing dishes on it due to the Inertia of rest.

Inertia of motion (kinetic inertia)

- The property by which it remains in state of motion

1. A person in running bus , falls forward the bus when stops suddenly.
2. Athlete taking a short run before a jump.

Forces on a book at rest on a horizontal surface



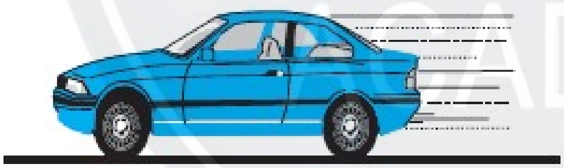
The two forces are

i) the force due to gravity (i.e., its weight *W*) acting downward

ii) the upward force on the book by the table, the normal force *R*.

- Since the book is rest the net force on the book must be zero.

Forces on a car



- When the car is stationary, there is no net force acting on it.
- During pick-up, it accelerates. It is the frictional force that accelerates the car as a whole.
- When the car moves with constant velocity, there is no net external force.
- Friction on the front wheels opposes the spinning, so friction must

point in the forward direction.

MOMENTUM (P)

• Momentum is the product of its mass and velocity

$$\vec{P} = m\vec{v}$$

• Momentum is a vector quantity

Some Situations relating momentum and applied force Situation -1

1. A much greater force is needed to push the truck than the car to bring them to the same speed in same time.
2. A greater opposing force is needed to stop a heavy body than a light body in the same time, if they are moving with the same speed.
3. If two stones, one light and the other heavy, are dropped from the top of a building, a person on the ground will find it easier to catch the light stone than the heavy stone.

Reason

- In these cases, change in momentum is greater for a heavy body.
- External force required is proportional to change in momentum for the given time.

Situation -2

1. A bullet fired by a gun can easily pierce human tissue before it stops, resulting in casualty.

2. The same bullet fired with moderate speed will not cause much damage

Reason

- Velocity is high for a bullet from a gun – the change in momentum is high
- External force required to stop the bullet is proportional to change in momentum for a given time.

Situation -3

i) A seasoned cricketer catches a cricket ball coming in with great speed far more easily than a novice, who can hurt his hands in the act

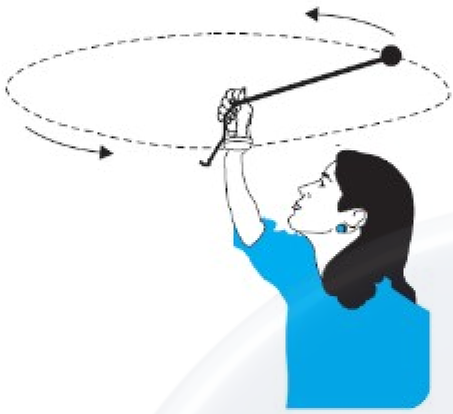
Reason

- External force depends on the time in which the momentum change is brought about.
- The change in momentum brought about in a shorter time needs greater applied force and vice versa.

Situation -4

- Suppose a stone is rotated with uniform speed in a horizontal plane by means of a string, the magnitude of momentum is fixed, but its direction changes
- The force needed to change in momentum is provided by our hand through the string.
- Our hand needs to exert a greater force if the stone is rotated at

greater speed or in a circle of smaller radius, or both



Reason

- External force is proportional to change in momentum.

NEWTON'S SECOND LAW OF MOTION

- **The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts.**

- That is

$$\vec{F} \propto \frac{\Delta \vec{p}}{\Delta t} \quad \text{or} \quad \vec{F} = k\frac{\Delta \vec{p}}{\Delta t}$$

- Where Δp – change in momentum in the time interval Δt and k – constant of proportionality.
- Taking the limit Δt→0,

$$\vec{F} = k\frac{d\vec{p}}{dt}$$

- For a body of fixed mass m,

$$\frac{d\vec{p}}{dt} = m\frac{dv}{dt} = m\vec{a}$$

- Thus $\vec{F} = km\vec{a}$
- The S I unit of force (newton) is defined such that k=1.
- Therefore

$$\vec{F} = m\vec{a}$$

- This law is applicable to both single particle and a system of particles.

Definition of newton

• One newton is that force, which causes an acceleration of 1m/s2, to a mass of 1kg.

$$1N = 1kgms^{-2}$$

Newton's First Law from Second Law

- We have *F* = *ma* ,
- Thus, when F=0 , a= 0- this is first law of motion.

Newton's second law in vector component form

- The second law of motion is a vector law. It is equivalent to three equations, one for each component of the vectors

$$F_x = \frac{dp_x}{dt} = ma_x$$

$$F_y = \frac{dp_y}{dt} = ma_y$$

$$F_z = \frac{dp_z}{dt} = m\,a_z$$

- Thus, if the force makes an angle with the velocity of a body , it changes only the component of velocity along the direction of force.
- The component of velocity normal to the force remains unchanged.

PROBLEM

• A bullet of mass 0.04 kg moving with a speed of 90 m s-1 enters a heavy wooden block and is stopped after a distance of 60 cm. What is the average resistive force exerted by the block on the bullet?

Solution

- Given m=0.04kg, v0 = 90 m/s , x= 0.6m, v=0
- The acceleration of the bullet is given by

$$v^2 = v_0{}^2 + 2ax$$
$$\Rightarrow 0 = 90^2 + 2 \times a \times 0.6$$
$$a = -\frac{90^2}{2 \times 0.6} = -6750 m/s$$

- The resistive force is

$$F = ma = 0.04 \times (-6750) = -270N$$

Impulse

- The product of force and time.

$\boldsymbol{Impulse = Force \times Time\ Duration}$ **= Change in Momentum**

$$I = F \times \Delta t = \Delta p$$

- Unit of impulse is **newton-second (Ns).**

Impulsive force

- A large force acting for a short time to produce a finite change in momentum.
- Examples are force when a ball hits on a wall, force exerted by a bat

on a ball, force on a nail by a hammer etc.

PROBLEM

• A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of 12 m s–1. If the mass of the ball is 0.15 kg, determine the impulse imparted to the ball. (Assume linear motion of the ball)

Solution

Impulse = Change in momentum

$$= 0.15 \times 12 - (-0.15 \times 12) = 3.6 Ns$$

NEWTON'S THIRD LAW OF MOTION

- **To every action, there is always an equal and opposite reaction**.
- **Forces always occur in pairs. Force on a body A by B is equal and opposite to the force on the body B by A.**
- **Action and reaction force occurs simultaneously.**
- Action and reaction forces act on different bodies, not on the same body – they do not cancel each other.

$$F_{AB} = -F_{BA}$$

Examples

- When a man **walks on earth** he exerts a force in the backward direction- Action The earth exerts an equal reaction on man in the forward direction. As a result, he moves in forward direction.

- When **a bird flies** it exerts a force on the air by its wings.

The air exerts a reaction force on the wings in the opposite direction. As a result, the bird flies.

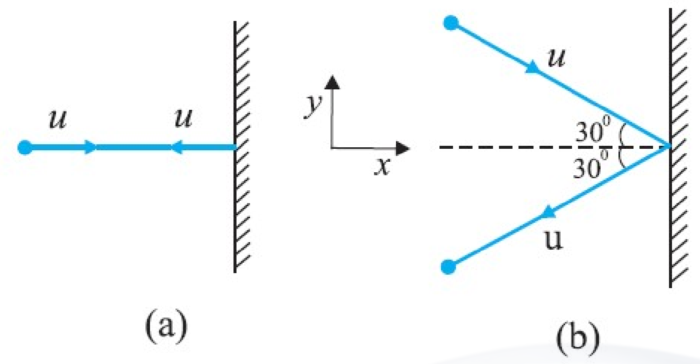
- When a **bullet is fired** from a gun , the force exerted on the bullet is action. The bullet exerts a reaction force on the gun in the opposite direction- recoil of gun
- In **a rocket t**he burnt gas at a very high-pressure escapes through the nozzle with a tremendous force.

This escaping gas exerts a reaction force on the rocket in the opposite direction. Thus, rocket moves forward.

PROBLEM

• Two identical billiard balls strike a rigid wall with the same speed but at different angles, and get reflected without any change in speed as shown in fig. What is

1. The direction of the force on the wall due to each ball?
2. the ratio of the magnitudes of impulses imparted to the balls by the wall?



Solution

i) To find the direction of force, impulse (change in momentum) of the ball is

calculated.

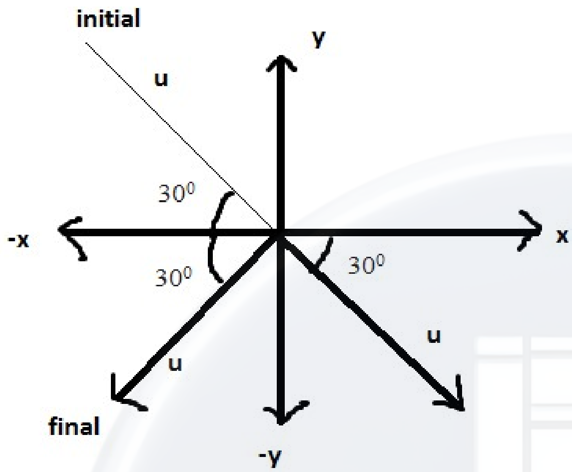
Case (a)

$$(p_x)_{initial} = mu$$
$$(p_x)_{final} = -mu$$
$$(p_y)_{initial} = 0$$
$$(p_y)_{final} = 0$$

- Therefore x – component of impulse = – 2mu
- y – component of impulse = 0
- Impulse and force are in the same direction
- Thus, the force on the ball due to the wall is normal to the wall, along the negative xdirection.
- From Newton’s Third law, the force on the wall due to the ball is

normal to the wall, along the positive x-direction.

Case (b)



$$(p_x)_{initial} = m\,u\cos 30^\circ,\ (p_y)_{initial} = -\ m\,u\sin 30^\circ$$

$$(p_x)_{final} = -\,m\,u\cos 30^\circ,\ (p_y)_{final} = -\ m\,u\sin 30^\circ$$

- Thus

x-component of impulse $= -2\ m\,u\ \cos\ 30^\circ$
y-component of impulse $= 0$

- Therefore, the force on the wall due to the ball is normal to the wall, along the positive xdirection.

ii) The ratio of the magnitudes of the impulses imparted to the balls is

$$2mu/\left(2mu\cos 30^{\circ}\right) = \frac{2}{\sqrt{3}} \approx 1.2$$

THE LAW OF CONSERVATION OF MOMENTUM

- **The total momentum of an isolated system (a system with no external force) of interacting particles is conserved.**
- **From Newton's second law** *dp*

$$F = \frac{dp}{dt}$$

When F=0, we get

$$F = \frac{dp}{dt} = 0$$

$$dp = 0$$

$$\therefore\ p = \text{constant}$$

- **Therefore, when F=0, initial momentum = final momentum.**

Applications of conservation of momentum Recoil of a gun

- Velocity of a bullet- muzzle velocity
- Movement of gun backward, when a bullet is fired- recoil of gun
- According to conservation of momentum momentum before firing =

momentum after

firing

- Thus

$$0 = mu + MV$$

Where m- mass of bullet, u- velocity of bullet, M- mass of gun, V- recoil velocity of gun

- Therefore

$$V = \frac{-mu}{M}$$

- The negative sign shows that velocity of gun is opposite to that of bullet
- Recoil velocity is very small (since M > m)

Rocket propulsion

- When a rocket is fired, fuel is burnt in the combustion chamber.
- The hot gas at very high-pressure escapes through the nozzle with a very high velocity

- The escaping gas has a very high momentum
- In order to conserve momentum, the rocket moves in the forward direction.

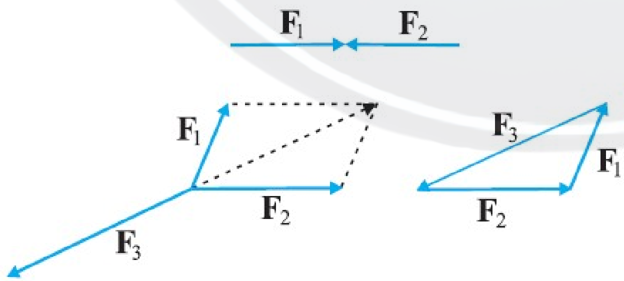
EQUILIBRIUM OF A PARTICLE

- Equilibrium of a particle in mechanics refers to the situation when the net external force on the particle is zero.
- The forces acting at a point are called **concurrent forces**
- If two forces **F1** and **F**2, act on a particle, equilibrium requires

$$\mathbf{F}_1 = -\mathbf{F}_2$$

- Equilibrium under three concurrent forces **F**1, **F**2 and **F**3 requires that

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$$



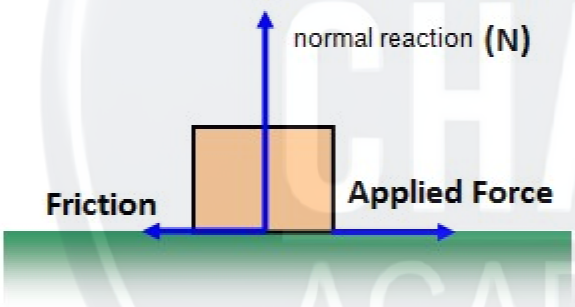
COMMON FORCES IN MECHANICS

i) Gravitational force –

- Every object on the earth experiences the force of gravity due to the earth.

ii) Contact forces

- A contact force on an object arises due to contact with some other object: solid or fluid.
- **When bodies are in contact there are mutual contact forces (for each pair of bodies) satisfying the third law.**
- The component of contact force normal to the surfaces in contact is called **normal reaction**.
- The component parallel to the surfaces in contact is called **friction.**



iii) Force due to spring

- When a spring is compressed or extended by an external force, a restoring force is generated.
- This force is usually proportional to the compression or elongation (for small displacements).
- The spring force *F is written as* **F = – k x** *where x is the* displacement and *k is the force constant.*
- The negative sign denotes that the force is opposite to the

displacement from the unstretched state.

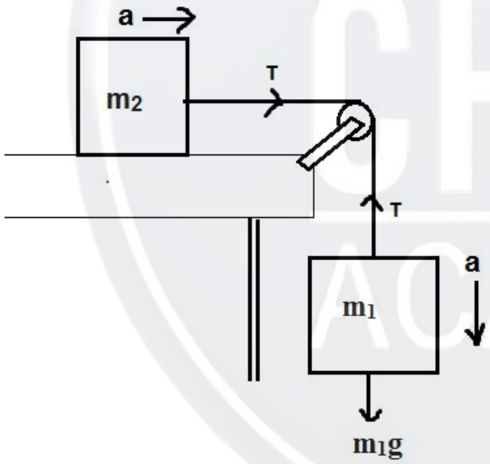
iv) Tension in a string (T)

- The restoring force in a string is called tension.
- The direction is always away from the body

The different contact forces of mechanics mentioned above fundamentally arise from electrical forces.

MOTION OF CONNECTED BODIES

Body moving on a horizontal frictionless table



- The net force on m_1 is

$$m_1 g - T = m_1 a$$

- The net force on m_2 is

$$T = m_2 a$$

- Adding two equations

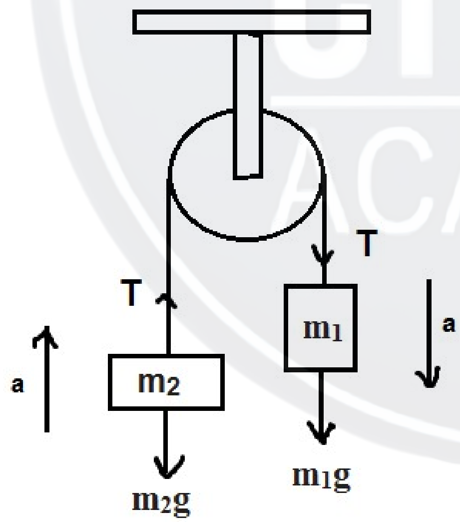
$$m_1 g = (m_1 + m_2)a$$

$$a = \frac{m_1 g}{(m_1 + m_2)}$$

- And

$$T = m_2 a = \frac{m_1 m_2}{(m_1 + m_2)} g$$

Motion in a frictionless pulley



- The net force on m_1 is

$$m_1 g - T = m_1 a$$

- The net force on m_2 is

$$T - m_2 g = m_2 a$$

- Adding two equations

$$m_1 g - m_2 g = (m_1 + m_2)a$$

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)}$$

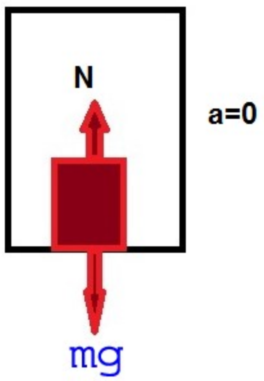
- And

$$T = m_2 g + m_2 a = m_2 g + m_2 \frac{m_1 - m_2}{(m_1 + m_2)} g$$

$$T = \frac{2m_1 m_2}{(m_1 + m_2)} g$$

Motion in a lift

a) Lift at rest or moving with constant velocity upward or downward

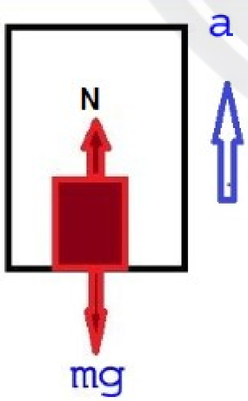


- Here acceleration a=0
- The net force is

$$N - mg = ma = 0$$
$$\therefore N = mg$$

- Thus apparent weight = Actual weight

b) Lift moves up with acceleration *a*

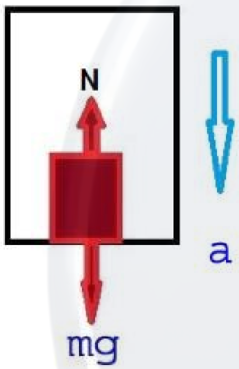


- **The net force**

$$N - mg = ma$$
$$\therefore N = m(g + a)$$

- Apparent weight > Actual weight

c) Lift moves down with acceleration *a*

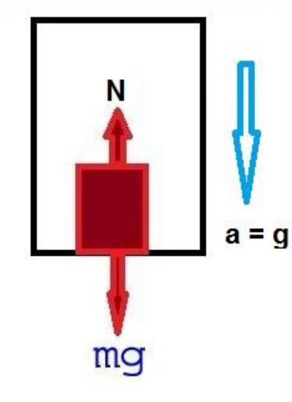


- The net force

$$mg - N = ma$$
$$\therefore N = m(g - a)$$

- Apparent weight < Actual weight

d) Lift falls down freely



- Here a= g
  $\therefore N = m(g-g) = o$
- Apparent weight = 0

FRICTION

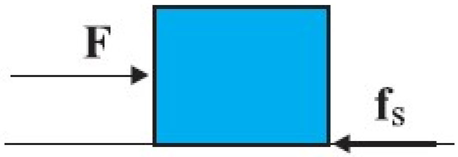
- ***Friction*** is the force which opposes the relative motion between two surfaces in contact.
- It acts ***tangential*** to the surface of contact.
- Arises due to
1. adhesive force between surfaces
2. irregularities of plane surface
- There are two types

i) Static friction

ii) Kinetic friction

Static friction

- Friction between two surfaces in contact as long as the bodies is at rest.



- Its value increases from zero to a maximum value called limiting friction (fsmax).
- Limiting friction is the static frictional force just before sliding.

Laws of Static Friction

- The *magnitude of limiting friction* is *independent of area* of the contact between the bodies.
- The limiting friction is proportional to the normal reaction N.

$$f_s^{\max} \propto N$$
$$f_s^{\max} = \mu_s N$$

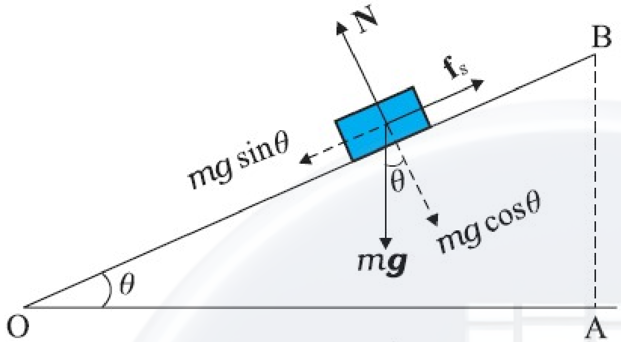
- μ_s - coefficient of static friction.

$$\mu_s = \frac{f_s^{\max}}{N}$$

- Thus , value of static friction may be written as

$$f_s \le \mu_s N$$

Angle of friction (θ)



- The angle at which the body begin to slide on an inclined plane is called **angle of limiting static friction or angle of repose**
- The weight of the body can be resolved in to two components.
- Just before sliding

$$mg\sin\theta = f_s^{\max}$$
$$mg\cos\theta = N$$

- Dividing the two equations

$$\frac{mg\sin\theta}{mg\cos\theta} = \frac{f_s^{\max}}{N}$$
$$\tan\theta = \frac{f_s^{\max}}{N} = \mu_s$$

- Thus, coefficient of static friction is the tangent of the angle of limiting friction.

PROBLEM-1

• Determine the maximum acceleration of the train in which a box lying on its floor will remain stationary, given that the co-efficient of static friction between the box and the train's floor is 0.15.

Solution

• Since the acceleration of the box is due to the static friction,

$$ma = f_s \le \mu_s N = \mu_s mg$$
$$a \le \mu_s g$$
$$a_{\max} = \mu_s g = 0.15 \times 10 m/s^2 = 1.5 m/s^2$$

PROBLEM-2

- A mass of 4 kg rests on a horizontal plane. The plane is gradually inclined until at an angle = 15° with the horizontal, the mass just begins to slide. What is the coefficient of static friction between the block and the surface?

Solution

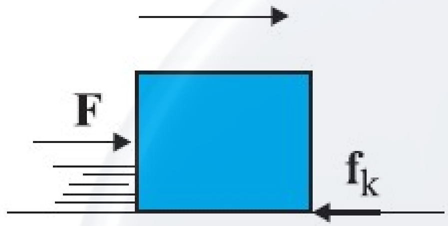
- We have

$$\tan\theta = \mu_s$$

$$\mu_s = \tan 15^0 = 0.27$$

Kinetic friction

- Friction experienced by a body when it moves



- Two types:

1. ***Sliding friction***
2. ***Rolling friction***

- ***Rolling friction < sliding friction < static friction***

Laws of kinetic friction

- *Kinetic friction does not depend on the nature of the two surfaces in contact.*
- *Kinetic friction is proportional to the normal reaction.*

$$f_k \propto N$$
$$f_k = \mu_k N$$

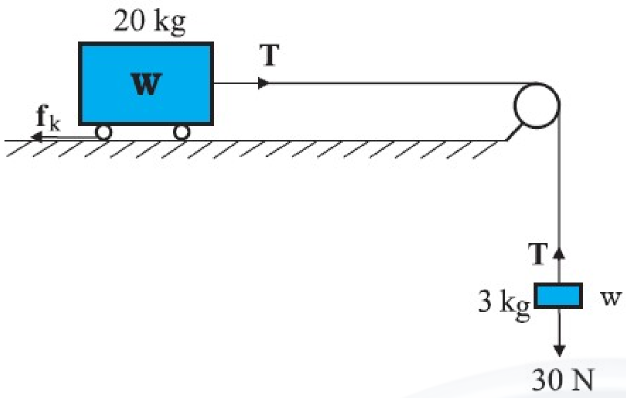
- μk is the coefficient of kinetic friction
- Coefficient of kinetic friction is less than that of static friction

Rolling friction

- Friction when a body rolls on a surface
- Very small compared to sliding friction-surface area of contact is small
- Advantage of Rolling friction is made use in ball bearings

PROBLEM

• What is the acceleration of the block and trolley system shown in the figure, if the coefficient of kinetic friction between the trolley and the surface is 0.04? What is the tension in the string? (Take g = 10 m s-2). Neglect the mass of the string.



Solution

- Net force on 2kg mass is
  $30 - T = 2a$, a –acceleration
- Net force on trolley is
  $T - f_k = 20a$
- Now $f_k = \mu_k N$
  μ_k=0.04,
  N= 20 x 10= 200 N
- Thus
  $T - 0.04 \times 200 = 20a$
  $T - 8 = 20a$
- Solving the equations , we get
  a=22/23 =0.96m/s^2 and T = 27.1 N

FRICTION AS A NECESSARY EVIL

• Friction is considered as a **necessary evil**, because it has both advantages and disadvantages.

Advantages of friction

- We are able to walk on the ground due to friction
- We can hold an object in hand due to friction
- Meteors burn in air due to friction.

Disadvantages of friction

- When a vehicle moves lot of energy is lost to overcome friction
- Excess heat produced in machines causes wear and tear to parts
- Atmospheric friction is disadvantageous to rockets and satellites

Ways to minimize friction

- Using lubricants like, grease, oil, wax etc.
- Using ball bearings or roll bearings
- Using anti-friction metals or alloys
- Separating the surfaces by an air cushion • Streamlining the body of vehicles
- Polishing the surfaces.

CIRCULAR MOTION

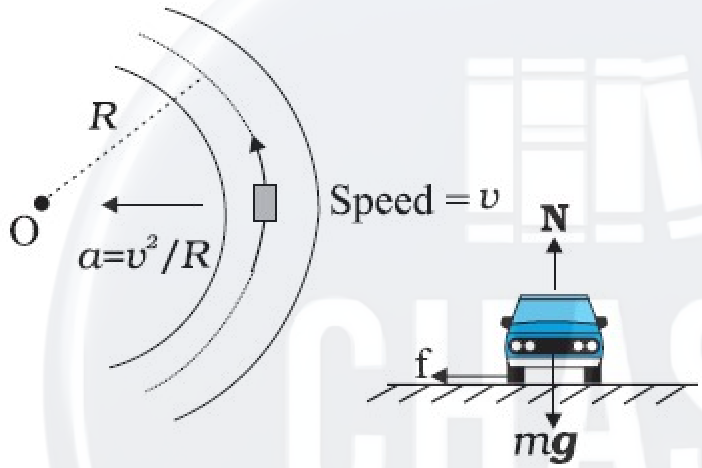
- Acceleration of a body moving in a circle of radius R with uniform speed v is v2/R directed towards the centre.
- According to the second law, the force providing this acceleration is

$$f_C = \frac{mv^2}{R}$$

- This force directed forwards the centre is called the **centripetal force**.

- For a stone rotated in a circle by a string, the centripetal force is provided by the tension in the string.
- The centripetal force for motion of a planet around the sun is the gravitational force on the planet due to the sun.
- For a car taking a circular turn on a horizontal road, the centripetal force is the force of friction.

Motion of a car on a level road



Three forces act on the car

1. **The weight of the car, *mg***
2. **Normal reaction, *N***
3. **Frictional force, *f***

Maximum speed of the car on a level road

- As there is no acceleration in the vertical direction

$$N - mg = 0$$
$$N = mg$$

- The centripetal force required for the circular motion is provided by the frictional force between road and the car tyres.
- Thus

$$f \leq \mu_s N = \frac{mv^2}{R}$$

$$\therefore v^2 \leq \frac{\mu_s RN}{m} = \frac{\mu_s Rmg}{m}$$
$$v^2 \leq \mu_s Rg$$

- Thus, for a given value of μs and R, the maximum speed of circular motion of the car is given by

$$v_{\max} = \sqrt{\mu_s Rg}$$

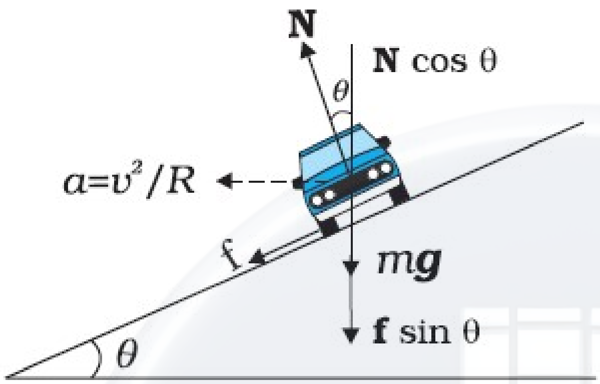
Motion of a car on a banked road

Banking of roads

- The phenomenon of raising outer edge of the curved road above the inner edge is called banking of roads.
- We can reduce the contribution of friction to the circular motion of the

car if the road is banked

Forces on a car in a banked road



Maximum possible speed of a car on a banked road – with friction

- Since there is no acceleration along the vertical direction, the net force along this direction must be zero.
- Thus

$$N\cos\theta = mg + f\sin\theta$$

- The centripetal force is provided by the horizontal components of *N and f.*

$$N\sin\theta + f\cos\theta = \frac{mv^2}{R}$$

- But for maximum speed , v_{max}, $f = \mu_s N$
- Thus

$$N\cos\theta = mg + \mu_s N\sin\theta$$

$$N(\cos\theta - \mu_s \sin\theta) = mg$$

$$N = \frac{mg}{(\cos\theta - \mu_s \sin\theta)}$$

- Also

$$N(\sin\theta + \mu_s \cos\theta) = \frac{mv_{max}^2}{R}$$

- Substituting for N in this equation we get,

$$\frac{mg(\sin\theta + \mu_s \cos\theta)}{(\cos\theta - \mu_s \sin\theta)} = \frac{mv_{max}^2}{R}$$

$$v_{max}^2 = \frac{Rg(\sin\theta + \mu_s \cos\theta)}{(\cos\theta - \mu_s \sin\theta)}$$

- Therefore

$$v_{max} = \sqrt{\frac{Rg(\sin\theta + \mu_s \cos\theta)}{(\cos\theta - \mu_s \sin\theta)}}$$

  - Dividing numerator and denominator by cosθ, we get

$$v_{max} = \sqrt{\frac{Rg(\tan\theta + \mu_s)}{(1 - \mu_s \tan\theta)}}$$

  - Thus, maximum possible speed of a car on a banked road is greater than that on a flat road.

Speed of the car – without friction

- If there is no friction, $\mu_s=0$,therefore the speed of the car is

$$v_0 = \sqrt{Rg \tan\theta}$$

- This is called the **optimum speed.**
- At this speed, frictional force is not needed to provide the necessary centripetal force.
- Driving at this speed on a banked road will cause little wear and tear of the tyres.

PROBLEM-1

- A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The coefficient of static friction between the tyres and the road is 0.1 Will the cyclist slip while taking the turn ?

Solution

- The condition for the cyclist not to slip is given by

$$v^2 \le \mu_s Rg$$

- Given , R = 3 m, g = 9.8 m s^{-2}, μ_s = 0.1. v = 18 km/h = 5 m s^{-1}

$$v^2 = \mu_s Rg = 0.1 \times 3 \times 9.8 = 2.94 m^2 / s^2$$

- Here v^2 = 25, the condition not obeyed.
- The cyclist will slip while taking the circular turn.

PROBLEM -2

- A circular racetrack of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race-car and the road is 0.2, what is the (a) optimum speed of the racecar to avoid wear and tear on its tyres, and (b) maximum permissible speed to avoid slipping ?

Solution

- Given , =150 ,μs =0.2, R=300m, g =9.8m/s2

a) Optimum speed is

$$v_0 = \sqrt{Rg \tan\theta} = \sqrt{300 \times 9.8 \times \tan 15^0} = 28.1 m/s$$

Maximum speed is

$$v_{\max} = \sqrt{\frac{Rg(\tan\theta + \mu_s)}{(1 - \mu_s \tan\theta)}} = 38.1 m/s$$