

## Chapter 15 Waves

### Wave Motion

- The motion of a disturbance from one point to another by the vibrations of the particles of the medium about their mean position is known as wave motion.
- It is a mode of transfer of energy from one point to another.
- The waves are mainly of three types: (a) mechanical waves, (b) electromagnetic waves and (c) matter waves.

### Mechanical waves

- Exist only within a material medium, such as water, air, and rock
- examples : – water waves, sound waves, seismic waves, etc
- two types : –

1) transverse waves

2) longitudinal waves

### Electromagnetic waves

- The electromagnetic waves do not require any medium for their propagation.
- All electromagnetic waves travel through vacuum at the same speed  $c$ , given by  $c = 299,792,458 \text{ m s}^{-1}$
- Examples of electromagnetic waves are visible and ultraviolet light, radio waves, microwaves, x-rays etc.

### Matter Waves

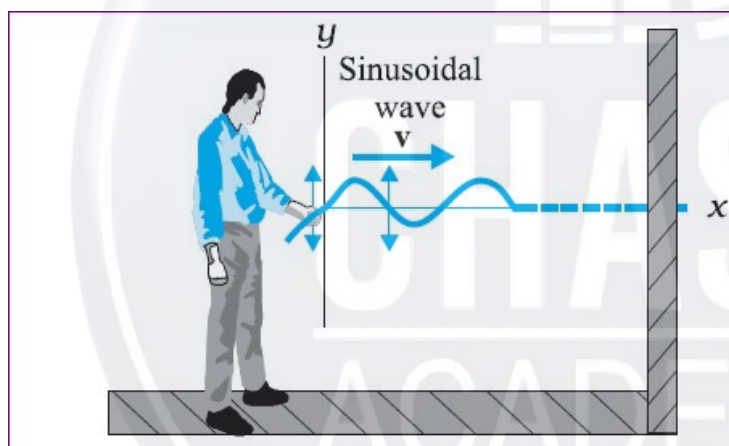




- Matter waves are associated with moving electrons, protons, neutrons and other fundamental particles, and even atoms and molecules
- Matter waves associated with electrons are employed in electron microscopes

### Transverse Waves

- In transverse waves, the constituents of the medium oscillate perpendicular to the direction of wave propagation.
- A point of maximum positive displacement in a wave is called crest, and a point of maximum negative displacement is called trough.

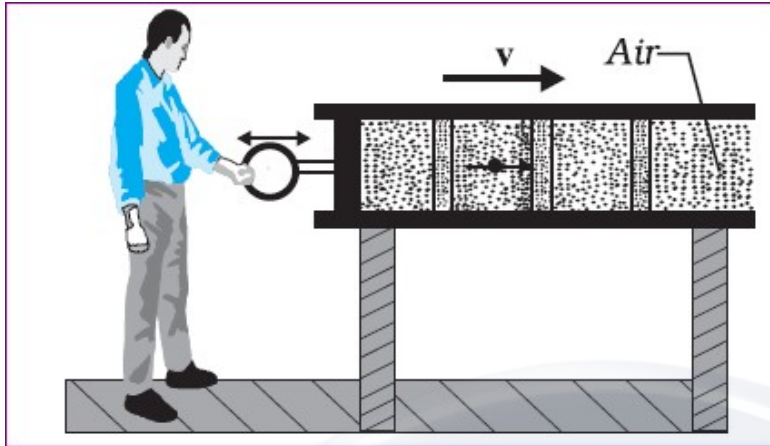


- Transverse waves can be propagated only through solids and strings, and not in fluids.

### Longitudinal Waves

- In longitudinal waves the constituents of the medium oscillate along the direction of wave propagation.





- Longitudinal sound waves propagate as compressions (high pressure region) and rarefactions (low pressure regions)
- longitudinal waves can propagate in all elastic media (solids and fluids)
- transverse and longitudinal waves travel with different speeds in the same medium.

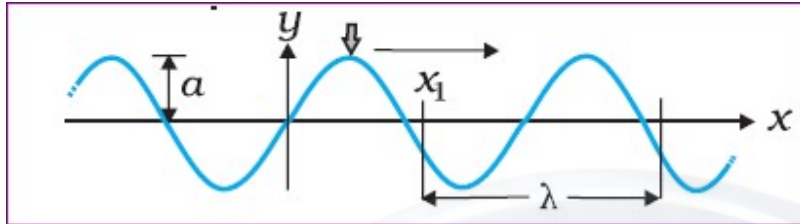
The waves on the surface of water

- The waves on the surface of water are of two kinds: capillary waves and gravity waves.
- Capillary waves are ripples of short wavelength.
- The restoring force that produces capillary waves is the surface tension of water.
- Gravity waves have wavelengths typically ranging from several metres to several hundred metres.
- The restoring force that produces gravity waves is the pull of gravity, which tends to keep the water surface at its lowest level.
- The waves in an ocean are a combination of both longitudinal and transverse waves.

Travelling or progressive wave



- A wave which travels from one point of the medium to another is called a travelling wave.



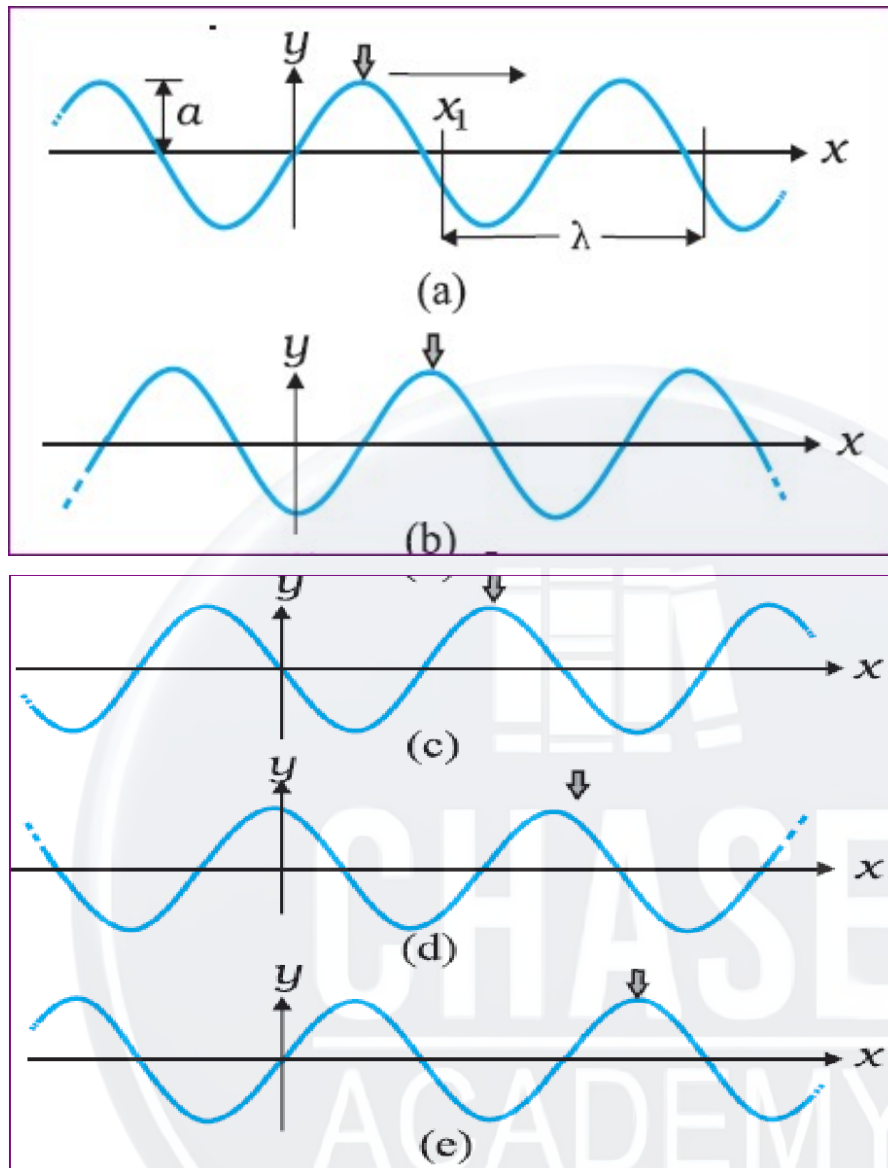
### Displacement Relation in a Progressive Wave

- At any time  $t$ , the displacement of a wave travelling in positive  $x$ -axis is given by

$$y(x, t) = a \sin(kx - \omega t + \phi)$$

- Where ,  $a$ - amplitude ,  $k$ - angular wave number or propagation constant ,  $\omega$ - angular frequency ,  $\phi$ - initial phase angle and  $(kx - \omega t + \phi)$  – phase Plots for a wave travelling in the positive direction of an  $x$ -axis at different values of time.





- A wave travelling in the negative direction of x-axis can be represented by

$$y(x, t) = a \sin (kx + \omega t + \phi)$$

Amplitude



- The amplitude  $a$  of a wave is the magnitude of the maximum displacement of the elements from their equilibrium positions as the wave passes through them.
- It is a positive quantity, even if the displacement is negative.

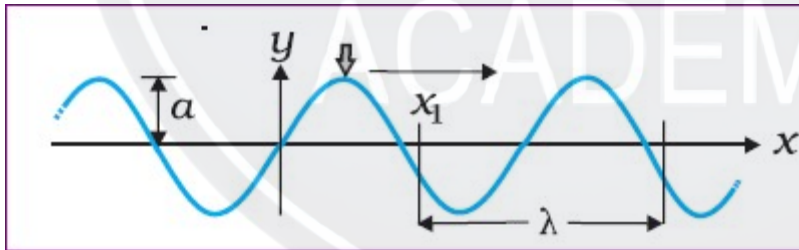
### Phase

- It describes the state of motion as the wave sweeps through a string element at a particular position  $x$
- The constant  $\phi$  is called the initial phase angle.

The value of  $\phi$  is determined by the initial ( $t = 0$ ) displacement and velocity of the element (say, at  $x = 0$ ).

### Wavelength ( $\lambda$ )

- It is the minimum distance between two consecutive troughs or crests or two consecutive points in the same phase of wave motion.



### Propagation constant or the angular wave number ( $k$ )

- For  $t = 0$  and  $\phi = 0$



$$y(x, 0) = a \sin kx$$

- By definition, the displacement  $y$  is same at both ends of this wavelength, that is at  $x = x_1$  and at  $x = x_1 + \lambda$ .
- Thus

$$\begin{aligned} a \sin kx_1 &= a \sin k(x_1 + \lambda) \\ &= a \sin (kx_1 + k\lambda) \end{aligned}$$

- This condition can be satisfied only when,

$$k\lambda = 2\pi n$$

- where  $n = 1, 2, 3, \dots$ . Since  $\lambda$  is defined as the least distance between points with the same phase,  $n = 1$  and therefore

$$k = \frac{2\pi}{\lambda}$$

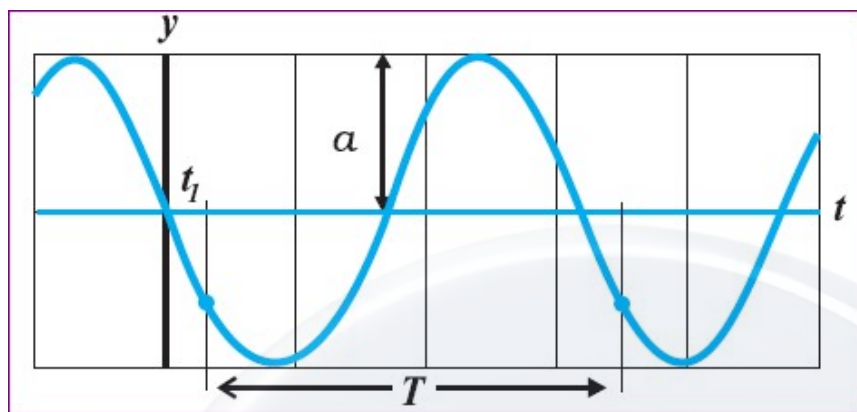
- $k$  is called the propagation constant or the angular wave number ; its SI unit is radian per metre or  $\text{rad m}^{-1}$

### Period

- The period of oscillation  $T$  of a wave is the time any string element



takes to move through one complete oscillation.



### Angular Frequency

- The angular frequency of the wave is given by

$$\omega = 2\pi/T$$

- Its SI unit is rad s<sup>-1</sup>

### Frequency

- It is the number of oscillations per unit time made by a string element as the wave passes through it
- The frequency  $\nu$  of a wave is defined as  $1/T$  and is related to the angular frequency  $\omega$  by



$$\nu = \frac{1}{T} = \frac{\omega}{2\pi}$$

- It is usually measured in hertz

### Displacement relation of a longitudinal wave

- In a longitudinal wave, the displacement of an element of the medium is parallel to the direction of propagation of the wave.
- The displacement function for a longitudinal wave is written as,

$$s(x, t) = a \sin (kx - \omega t + \phi)$$

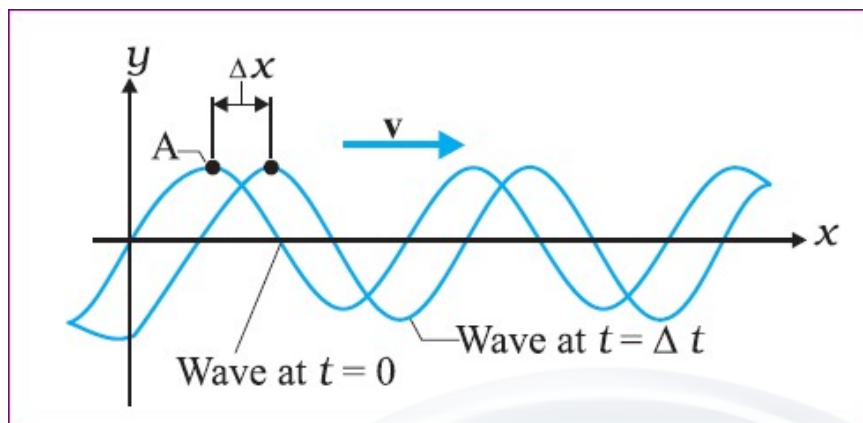
- where  $s(x, t)$  is the displacement of an element of the medium in the direction of propagation of the wave at position  $x$  and time  $t$ .

### The Speed of a Travelling Wave

- The speed of a wave is related to its wavelength and frequency by the relation

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda \nu$$





- The speed is determined by the properties of the medium.

### Speed of a Transverse Wave on Stretched String

- The speed of transverse waves on a string is determined by two factors,

(i) the linear mass density or mass per unit length,  $\mu$ , and (ii) the tension  $T$ .

- The linear mass density,  $\mu$ , of a string is the mass  $m$  of the string divided by its length  $l$ . therefore its dimension is  $[ML^{-1}]$ .
- The tension  $T$  has the dimension of force  $[M L T^{-2}]$ .
- Let the speed  $v = C \mu^a T^b$ , where  $c$  is a dimensionless constant.
- Taking dimensions on both sides  $[M^0 L^1 T^{-1}] = [M^1 L^{-1}]^a [M L T^{-2}]^b$   
 $= [M^{a+b} L^{-a+b} T^{-2b}]$
- Equating the dimensions on both sides we get  $a+b = 0$  , therefore  $a=-b$ ,  $-a+b = 1$ , therefore  $2b=1$  or  $b= \frac{1}{2}$  and  $a= -\frac{1}{2}$
- Thus

$$v = C \mu^{-\frac{1}{2}} T^{\frac{1}{2}},$$



or

$$v = C \sqrt{\frac{T}{\mu}}$$

- It can be shown that  $C=1$ , therefore the speed of transverse waves on a stretched string is

$$v = \sqrt{\frac{T}{\mu}}$$

- The speed of a wave along a stretched ideal string depends only on the tension and the linear mass density of the string and does not depend on the frequency of the wave.

### Speed of a Longitudinal Wave – Speed of Sound

- In a longitudinal wave the constituents of the medium oscillate forward and backward in the direction of propagation of the wave.
- The sound waves travel in the form of compressions and rarefactions of small volume elements of air.
- The speed of sound waves depends on
  1. Bulk modulus,  $B$  and
  2. Density of the medium,  $\rho$
- Using dimensional analysis we may write

$$v = C B^a \rho^b$$

- Taking dimensions  $[M^0 L^1 T^{-1}] = [M L^{-1} T^{-2}]^a [M L^{-3}]^b = [M^{a+b} L^{-a-3b} T^{-2a}]$



bT-2a]

- Equating the dimensions on both sides we get

$a+b = 0$  , therefore  $a=-b$ ,  $-2a=-1$ ,  $a=1/2$  , therefore  $b=-1/2$

- Therefore

$$v = C \sqrt{\frac{B}{\rho}}$$

- where C is a dimensionless constant and can be shown to be unity.
- Thus, the speed of longitudinal waves in a medium is given by,

$$v = \sqrt{\frac{B}{\rho}}$$

- The speed of propagation of a longitudinal wave in a fluid therefore depends only on the bulk modulus and the density of the medium.
- The bulk modulus is given by

$$B = - \frac{\Delta P}{\Delta V/V}$$

- Here  $\Delta V/V$  is the fractional change in volume produced by a change



in pressure  $\Delta P$ .

Speed of sound wave in a material of a bar

- The speed of a longitudinal wave in the bar is given by,

$$v = \sqrt{\frac{Y}{\rho}}$$

- where Y is the Young's modulus of the material of the bar.

**Speed of sound in different media**

CHASE  
ACADEMY



Medium	Speed (m s <sup>-1</sup> )
<b>Gases</b>	
Air (0 °C)	331
Air (20 °C)	343
Helium	965
Hydrogen	1284
<b>Liquids</b>	
Water (0 °C)	1402
Water (20 °C)	1482
Seawater	1522
<b>Solids</b>	
Aluminium	6420
Copper	3560
Steel	5941
Granite	6000
Vulcanised Rubber	54

### Newton's Formula

- In the case of an ideal gas, the relation between pressure  $P$  and volume  $V$  is given by

$$PV = Nk_B T$$



- Therefore, for an isothermal change it follows that

$$V\Delta P + P\Delta V = 0$$

$$-\frac{\Delta P}{\Delta V/V} = P$$

- Thus  $B=P$
- Therefore, the speed of a longitudinal wave in an ideal gas is given by,

$$v = \sqrt{\frac{P}{\rho}}$$

- This relation was first given by Newton and is known as Newton's formula. Laplace correction According to Newton's formula for the speed of sound in a medium, we get for the speed of sound in air at STP,

$$v = \left[ \frac{1.01 \times 10^5 \text{ N m}^{-2}}{1.29 \text{ kg m}^{-3}} \right]^{1/2} = 280 \text{ m s}^{-1}$$

- This is about 15% smaller as compared to the experimental value of  $331 \text{ m s}^{-1}$
- Laplace pointed out that the pressure variations in the propagation of sound waves are adiabatic and not isothermal.
- For adiabatic processes the ideal gas satisfies the relation,



$$PV^\gamma = \text{constant}$$

$$\text{i.e. } \Delta(PV^\gamma) = 0$$

$$P\gamma V^{\gamma-1} \Delta V + V^\gamma \Delta P = 0$$

- Thus for an ideal gas the adiabatic bulk modulus is given by,

$$B_{ad} = -\frac{\Delta P}{\Delta V/V} = \gamma P$$

- where  $\gamma$  is the ratio of two specific heats,  $C_p/C_v$ .
- The speed of sound is, therefore, given by,

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

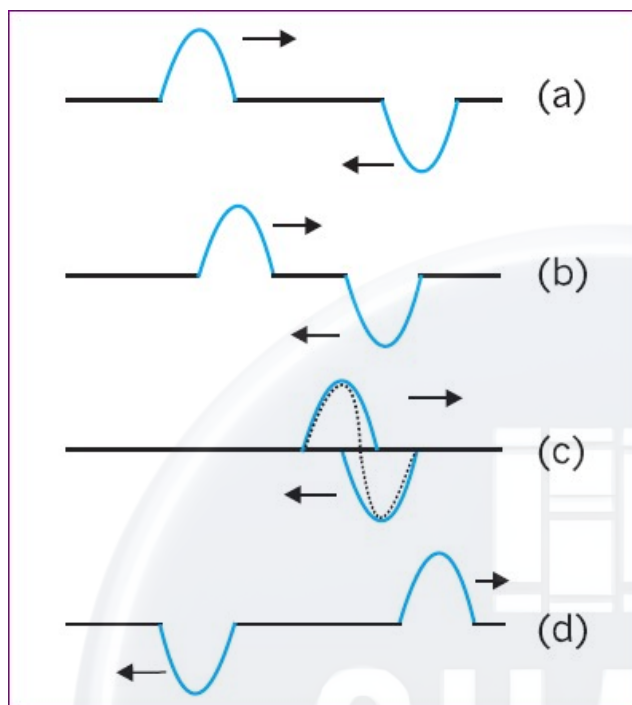
- This modification of Newton's formula is referred to as the Laplace correction.
- For air  $\gamma = 7/5$ , therefore the speed of sound in air at STP, we get a value  $331.3 \text{ m s}^{-1}$ , which agrees with the measured speed.

### The Principle of Superposition of Waves

- The principle of super position of waves states that the net displacement at a given time of a number of waves is the algebraic



sum of the displacements due to each wave.



- Let  $y_1(x, t)$  and  $y_2(x, t)$  be the displacements that any element of the string would experience if each wave travelled alone.
- The displacement  $y(x, t)$  of an element of the string when the waves overlap is then given by,

$$y(x, t) = y_1(x, t) + y_2(x, t)$$

- Let a wave travelling along a stretched string be given by,

$$y_1(x, t) = a \sin(kx - \omega t)$$



- And another wave, shifted from the first by a phase  $\phi$ ,

$$y_2(x, t) = a \sin(kx - \omega t + \phi)$$

- Both the waves have the same angular frequency, same angular wave number  $k$  (same wavelength) and the same amplitude  $a$ .
- Applying the superposition principle

$$y(x, t) = a \sin(kx - \omega t) + a \sin(kx - \omega t + \phi)$$

- Using the trigonometric relation

$$\omega = \sqrt{\frac{mgL}{I}}$$

$$y(x, t) = [2a \cos \frac{1}{2}\phi] \sin(kx - \omega t + \frac{1}{2}\phi)$$

- Thus, the resultant wave is also a sinusoidal wave, travelling in the positive direction of  $x$ -axis.
- The resultant wave differs from the constituent waves in two respects:

i) its phase angle is  $(\frac{1}{2})\phi$  and



II) its amplitude is the quantity given by

$$A(\phi) = 2a \cos (\frac{1}{2})\phi$$

- If  $\phi = 0$ , the amplitude of the resultant wave is  $2a$ , which is the largest possible value of  $A(\phi)$ .
- If  $\phi = \pi$ , the two waves are completely out of phase, the amplitude of the resultant reduces to zero.

## REFLECTION OF WAVES

- When a pulse or a travelling wave encounters a rigid boundary it gets reflected.
- If the boundary is not completely rigid or is an interface between two different elastic media, a part of the wave is reflected and a part is transmitted into the second medium.
- The incident and refracted waves obey Snell's law of refraction, and the incident and reflected waves obey the laws of reflection.
- A travelling wave, at a rigid boundary or a closed end, is reflected with a phase reversal.
- A travelling wave, at an open boundary is reflected without any phase change.
- Let the incident wave be represented by

$$y_i(x, t) = a \sin (kx - \omega t)$$

- then, for reflection at a rigid boundary the reflected wave is represented by,



$$\begin{aligned}y_r(x, t) &= a \sin (kx + \omega t + \pi) \\ &= -a \sin (kx + \omega t)\end{aligned}$$

- For reflection at an open boundary, the reflected wave is represented by

$$y_r(x, t) = a \sin (kx + \omega t)$$

### Standing Waves and Normal Modes

- The waveform or the disturbance does not move to either side is known as stationary wave or standing wave.
- Let the wave travelling in the positive direction of x-axis be

$$y_1(x, t) = a \sin (kx - \omega t)$$

- And the wave travelling in the negative direction of x-axis

$$y_2(x, t) = a \sin (kx + \omega t)$$

- The principle of superposition gives, for the combined wave



$$\begin{aligned}
 y(x, t) &= y_1(x, t) + y_2(x, t) \\
 &= a \sin(kx - \omega t) + a \sin(kx + \omega t) \\
 &= (2a \sin kx) \cos \omega t
 \end{aligned}$$

- The amplitude is zero for values of  $kx$  that give  $\sin kx = 0$ . Those values are given by

$$kx = n\pi, \text{ for } n = 0, 1, 2, 3, \dots$$

- Substituting  $k = 2\pi/\lambda$  in this equation, we get

$$x = n \frac{\lambda}{2}, \text{ for } n = 0, 1, 2, 3, \dots$$

### Nodes

- The positions of zero amplitude in a standing wave are called nodes.
- A distance of  $\lambda/2$  or half a wavelength separates two consecutive nodes.
- The amplitude has a maximum value of  $2a$ , which occurs for the values of  $kx$  that give  $|\sin kx| = 1$ .
- The values are

$$kx = (n + \frac{1}{2})\pi \text{ for } n = 0, 1, 2, 3, \dots$$



- Substituting  $k = 2\pi/\lambda$  in this equation, we get

$$x = (n + \frac{1}{2}) \frac{\lambda}{2} \text{ for } n = 0, 1, 2, 3, \dots$$

### Antinodes

- ♦ The positions of maximum amplitude are called antinodes.
- ♦ The antinodes are separated by  $\lambda/2$  and are located half way between pairs of nodes.

### Standing waves of Stretched Rings

- For a stretched string of length  $L$ , fixed at both ends, the two ends of the string have to be nodes.
- If one of the ends is chosen as position  $x = 0$ , then the other end is  $x = L$ . In order that this end is a node; the length  $L$  must satisfy the condition

$$L = n \frac{\lambda}{2}, \text{ for } n = 1, 2, 3, \dots$$

The standing waves on a string of length  $L$  have restricted wavelength given by

$$\lambda = \frac{2L}{n}, \text{ for } n = 1, 2, 3, \dots \text{ etc.}$$



- The frequencies corresponding to these wavelengths is given by

$$v = n \frac{v}{2L}, \text{ for } n = 1, 2, 3, \dots \text{ etc.}$$

- where  $v$  is the speed of travelling waves on the string.
- The set of frequencies possible in a standing wave are called the natural frequencies or modes of oscillation of the system.
- The frequency corresponding to  $n=1$  is

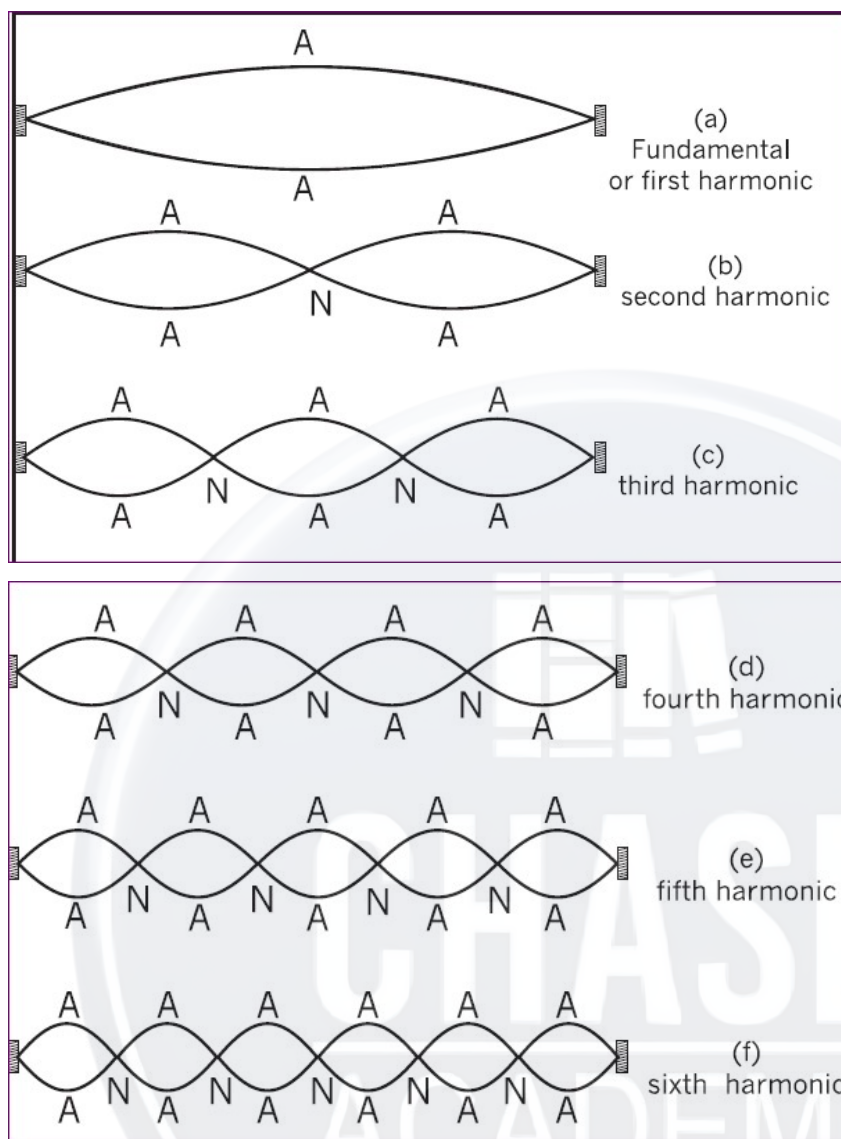
$$v = \frac{v}{2L}$$

- The oscillation mode with this lowest frequency ( $n=1$ ) is called the fundamental mode or the first harmonic.
- The second harmonic is the oscillation mode with  $n = 2$ . The third harmonic corresponds to  $n = 3$  and so on.

The frequencies associated with these modes are often labelled as  $v_1, v_2, v_3$  and so on.

- The collection of all possible modes is called the harmonic series and  $n$  is called the harmonic number.





### Modes of vibration of a pipe closed at one end

- In a closed pipe standing waves are formed such that a node at the closed end and antinode at open end.
- Now if the length of the air column is  $L$ , then the open end,  $x = L$ , is an antinode and therefore,



$$L = +\left(n\frac{1}{2} + \frac{\lambda}{2}\right)$$

- Where  $n=0,1,2,3,\dots$
- The modes, which satisfy the condition

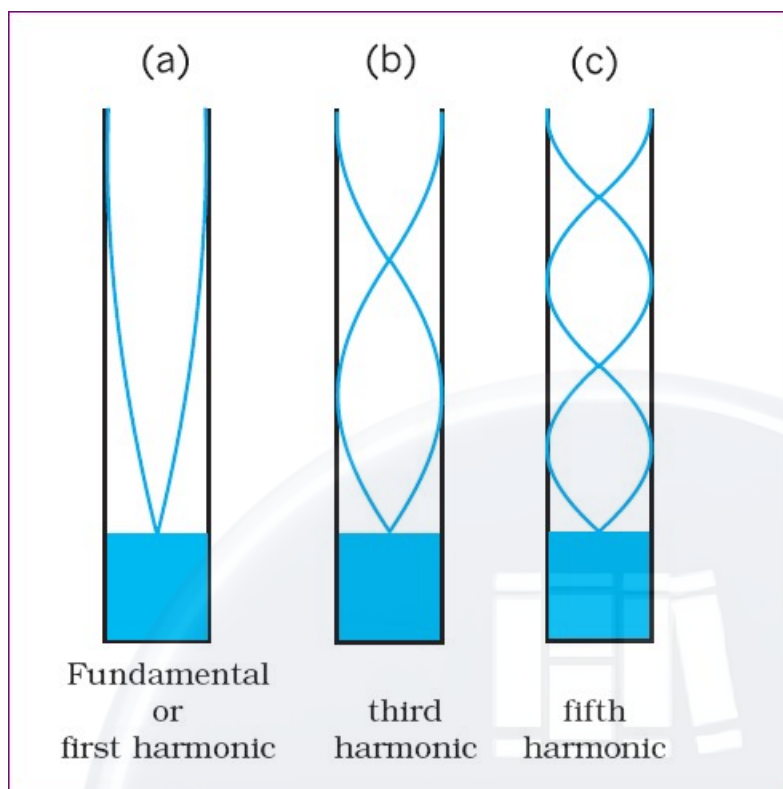
$$\lambda = \frac{2L}{\left(n + \frac{1}{2}\right)}, \text{ for } n = 0, 1, 2, 3, \dots$$

- The corresponding frequencies of various modes of such an air column are given by,

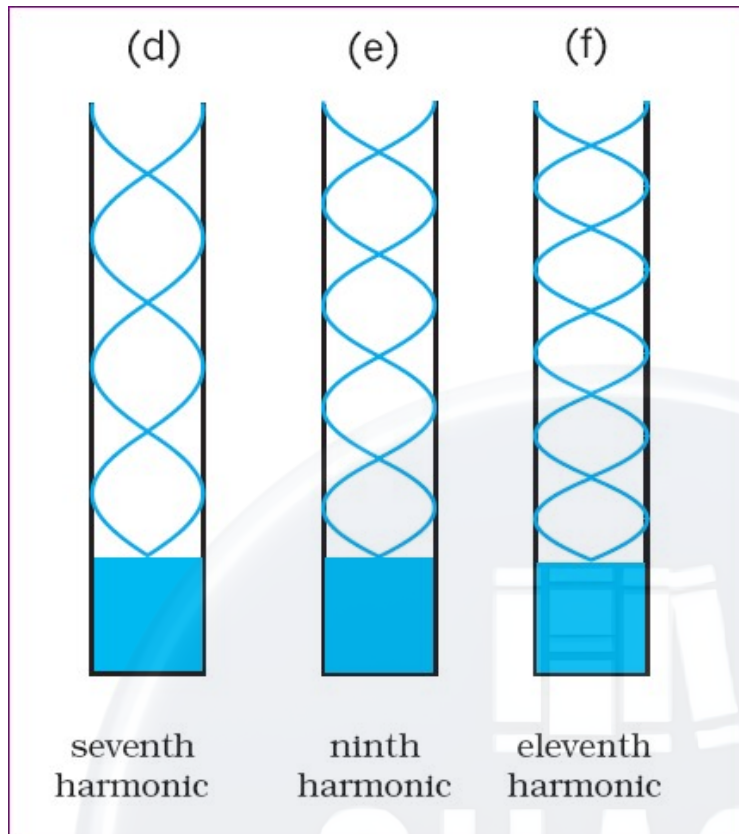
$$v = \left(n + \frac{1}{2}\right) \frac{v}{2L}, \text{ for } n = 0, 1, 2, 3, \dots$$

- The fundamental frequency is  $v/4L$  and the higher frequencies are odd harmonics of the fundamental frequency, i.e.  $3v/4L, 5v/4L, \dots$





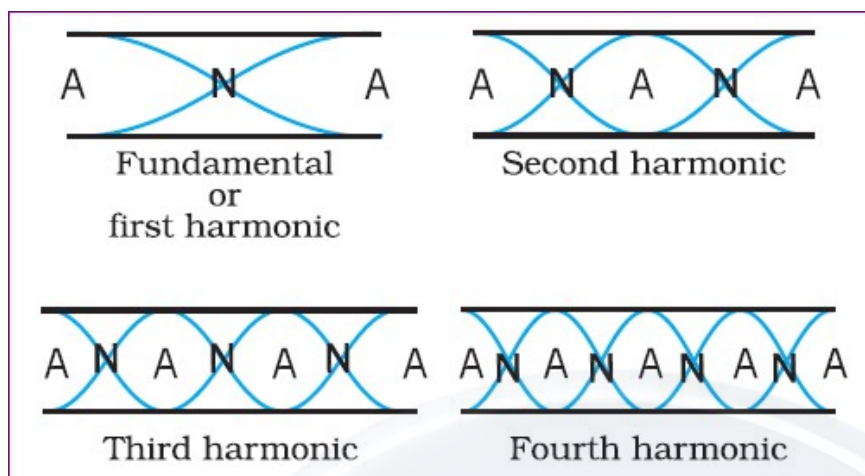




Pipe open at both ends

- In the case of a pipe open at both ends, there will be antinodes at both ends, and all harmonics will be generated.





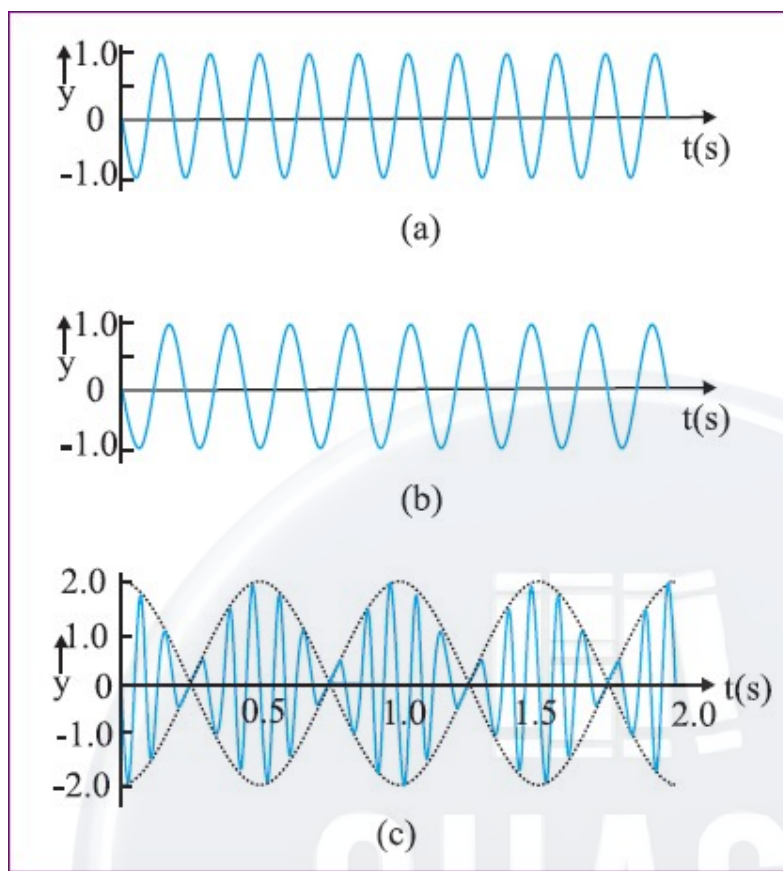
## BEATS

- The phenomenon of wavering of sound intensity when two waves of nearly same frequencies and amplitudes travelling in the same direction, are superimposed on each other is called beats.
- The beat frequency, is given by

$$v_{beat} = v_1 - v_2$$

The time-displacement graphs of two waves of frequency 11 Hz and 9 Hz



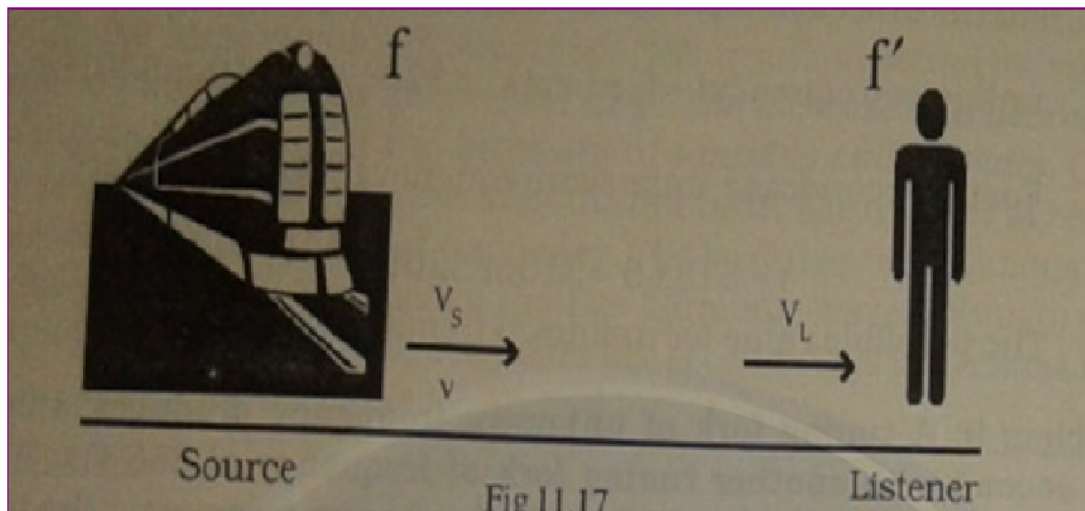


- Musicians use the beat phenomenon in tuning their instruments.
- If an instrument is sounded against a standard frequency and tuned until the beat disappears, then the instrument is in tune with that standard.

### Doppler Effect

- ***The apparent change in the pitch or the frequency of sound produced by a source due to relative motion of the source, listener or the medium is called Doppler effect.***





- It was proposed by Christian Doppler and tested experimentally by Buys Ballot
- All types of waves show Doppler effect.
- S- source
- $f$  – frequency of sound from source
- $V$  – velocity of sound
- $\lambda$ - wavelength

When Source and Listener at Rest

- When the source and the listener are at rest, the frequency of sound heard by the listener

$$f = \frac{V}{\lambda} \quad \text{or} \quad \lambda = \frac{V}{f}$$

When source and listener moving in the direction of sound

- The relative velocity of sound wave with respect to source is  $V - v_s$



- $V_s$  – velocity of source
- Thus, apparent wavelength is

$$\lambda' = \frac{V - V_s}{f}$$

- The relative velocity of sound with respect to listener is  $V' = -V - V_L$
- The apparent frequency of sound heard by the listener is

$$f' = \frac{V'}{\lambda'}$$

- Thus

$$f' = \frac{V - V_L}{V - V_s} = f \frac{(V - V_L)}{(V - V_s)}$$

Special cases

Source moving and listener stationary

a) Source moves towards the listener

- Now  $V_s = +ve$ ,  $V_L = 0$



- Thus  $f' = f \left( \frac{V}{V - V_s} \right)$

b) Source moves away from the listener

- Now  $V_s = -ve$  ,  $V_L = 0$

- Thus  $f' = f \left( \frac{V}{V + V_s} \right)$

Source stationary, listener moving

a) Listener moves towards the source

- Now  $V_L = -ve$  ,  $V_s = 0$

- Thus  $f' = f \left( \frac{V + V_L}{V} \right)$

b) Listener moves away from the source

- Now  $V_L = +ve$  ,  $V_s = 0$

- Thus  $f' = f \left( \frac{V - V_L}{V} \right)$



**Both source and listener moving****a) Source and listener move towards each other**

- Now  $V_S = +ve$ ,  $V_L = -ve$

- Thus  $f' = f \left( \frac{V - V_L}{V - V_S} \right)$

**b) Source and listener move away from each other**

- Now  $V_S = -ve$ ,  $V_L = +ve$

- Thus  $f' = f \left( \frac{V - V_L}{V + V_S} \right)$

**c) Source moves towards the listener and listener moves away**

- Now  $V_S = +ve$ ,  $V_L = +ve$

- Thus  $f' = f \left( \frac{V - V_L}{V - V_S} \right)$

1. Source moves away from the listener and listener moves towards the source

- Now  $V_S = -ve$ ,  $V_L = -ve$



- Thus  $f' = f \left( \frac{V + V_L}{V + V_S} \right)$

Effect of motion of the medium

- When the wind blows the air medium will move with a velocity  $w$

When wind moves towards the listener the velocity of sound is  $V + w$

- Thus, the apparent frequency

$$f' = f \left( \frac{(V + w) - V_L}{(V + w) - V_S} \right)$$

- If the wind is blowing from listener to the source, velocity of sound is  $V - w$

- Thus  $f' = f \left( \frac{(V - w) - V_L}{(V - w) - V_S} \right)$

Uses of Doppler Effect

- To estimate the speed of submarine, aero plane, automobile, etc
- To track artificial satellite
- To estimate velocity and rotation of star
- Doctors use it to study heart beats and blood flow in different parts of



the body. Here they use ultrasonic waves, and in common practice, it is called sonography.

- In the case of the heart, the picture generated is called echocardiogram.

